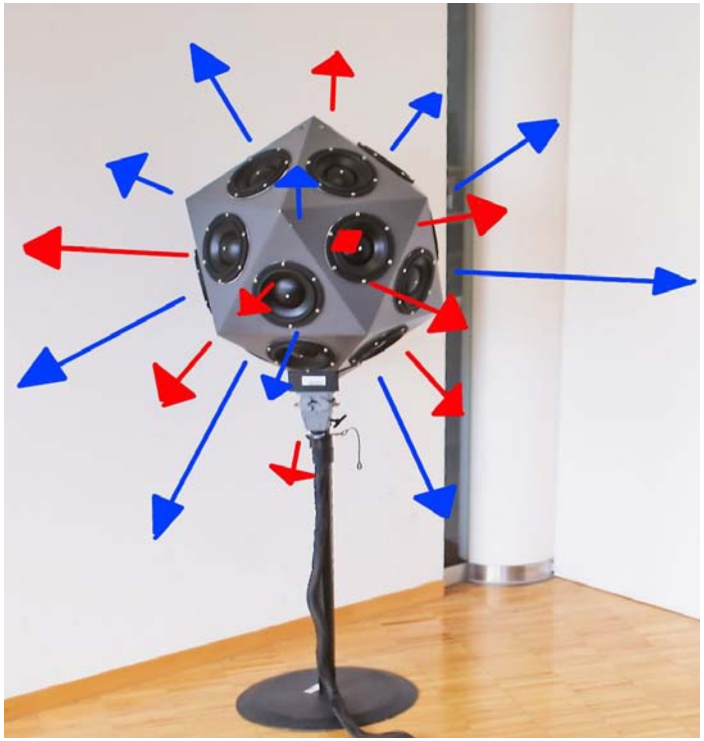
# Spherical Arrays for Sound-Radiation Analysis and Synthesis

Hannes Pomberger

Franz Zotter

# Spherical Arrays for Analysis and Synthesis of Radiation

Hohl, Deboy, Zotter, 2009
Baumgartner, Messner, 2010

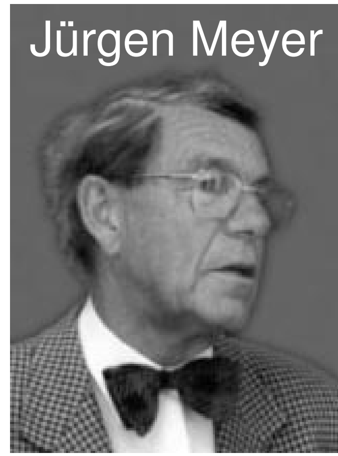


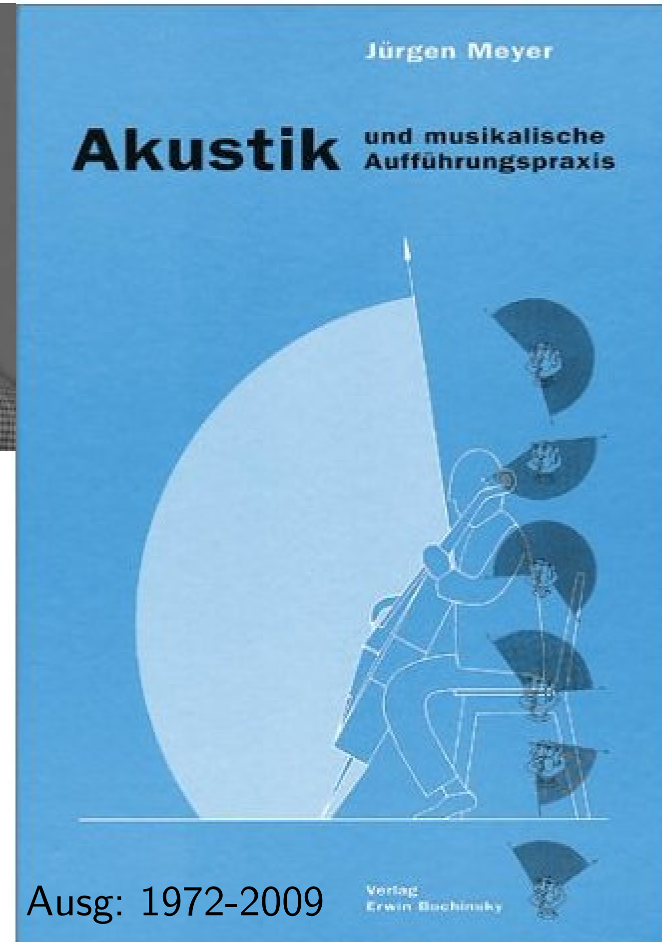
Zotter, Pomberger, 2007-2009
Zaar, Kößler 2009, Jochum, Reiner 2007

# Pioneering Works

Domains that are interested in sound-radiation of Musical Instruments:

- Recording
- Room Acoustics
- Music
- Perception




artificially excited instrument

1966: wind instruments.

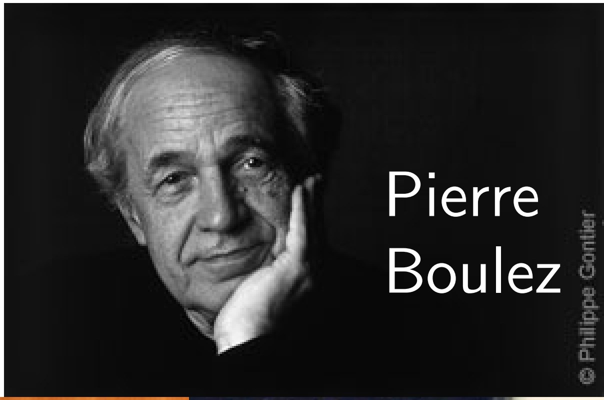
1972: 1st ed.

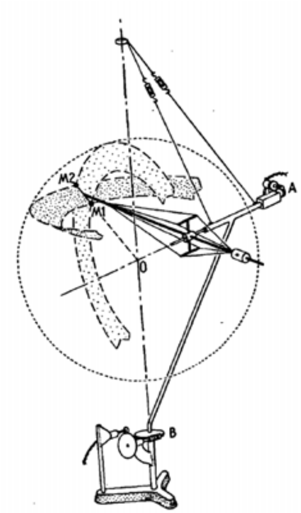
## Pioneering Works

- Jürgen Meyer (1972)
- Weinreich and Arnold (1980)

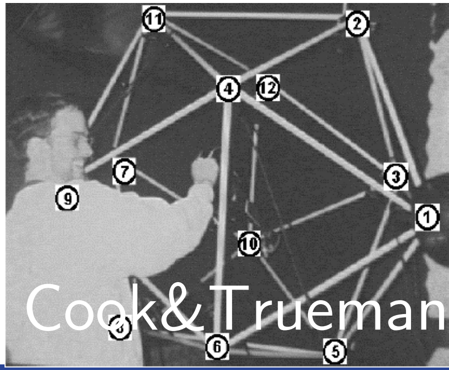
- IRCAM (70s, Pierre Boulez)
- Franck Giron (1996)
- Cook and Trueman (1998)

- Olivier Warusfel et al. (1997-)
  La Timée





## Remaining Issues

- Theoretical concept of sound-radiation recording and playback with spherical arrays

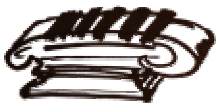
- Practical requirements, models, and processing ideas for sound-radiation

- Mathematical-physical basics for discrete recording, processing, and playback techniques

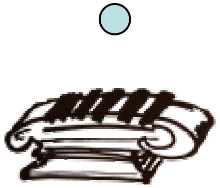
Note: Advantage of spherical microphone arrays
direct capture instruments under musical excitation

# Sound Radiation

- example:
  radiation „saron barung“
  (a Gamelan metallophone)



1st mic: frontal instrument height



2nd mic: frontal elevated

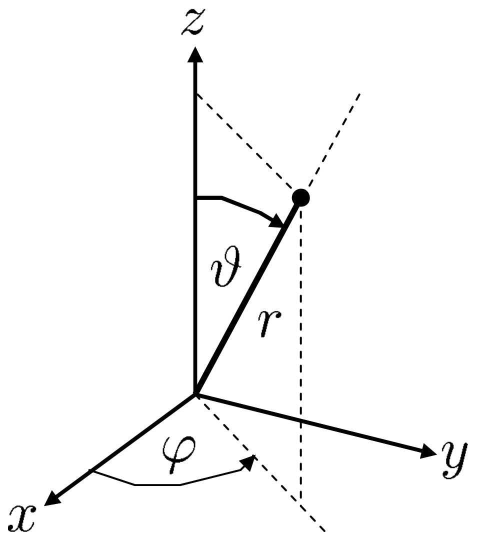
# Sound Radiation

- Sound-source captured at 2 positions: bonang barung

→ two signals $x_1(t)$ and $x_2(t)$

$x(\varphi, \vartheta, t)$ ...sound-radiation signal

$\varphi, \vartheta$ ...angular coordinates



# Theory of Sound-Radiation Analysis
## Soap-Bubble Model

- Radial sound particle velocity $x(\varphi,\vartheta,t) = v(\varphi,\vartheta,t)$

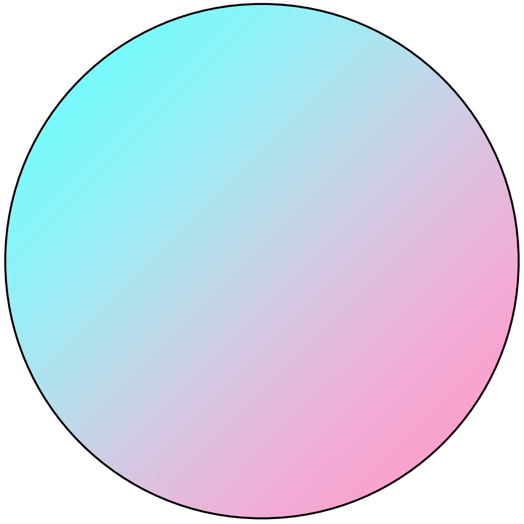


Image: put musician and instrument
into a big enough soap-bubble

# Theory of Sound-Radiation Analysis

## Soap-Bubble Model

- Radial sound particle velocity $x(\varphi, \vartheta, t) = v(\varphi, \vartheta, t)$

Image: put musician and instrument
into a big enough soap-bubble

# Theory of Sound-Radiation Analysis
## Surrounding Spherical Mic-Array

- Sound pressure

$$x(\varphi, \vartheta, t) = p(\varphi, \vartheta, t)$$

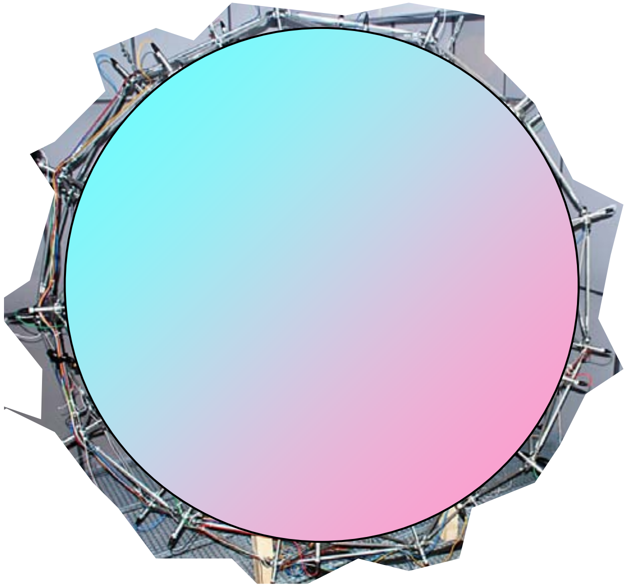


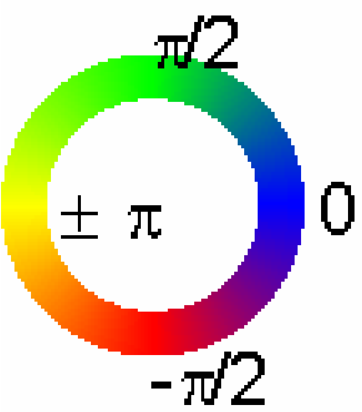
Image: put musician and instrument
into a big enough mic-array

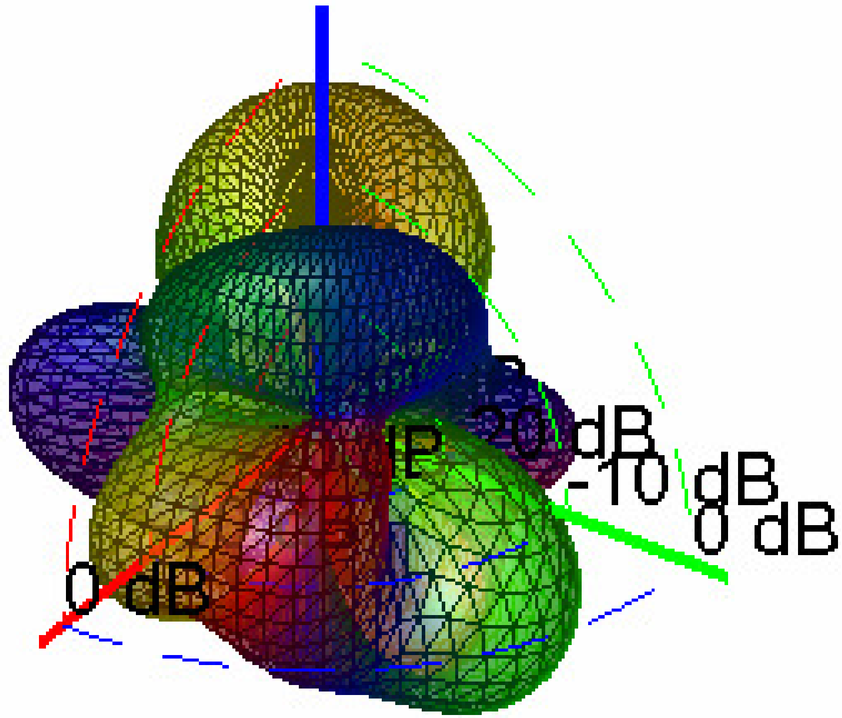
$$p(\varphi, \vartheta, t) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} Y_n^m(\varphi, \vartheta) \cdot \psi_{nm}(t)$$

# Spherical Radiation Pattern

angular band-limit

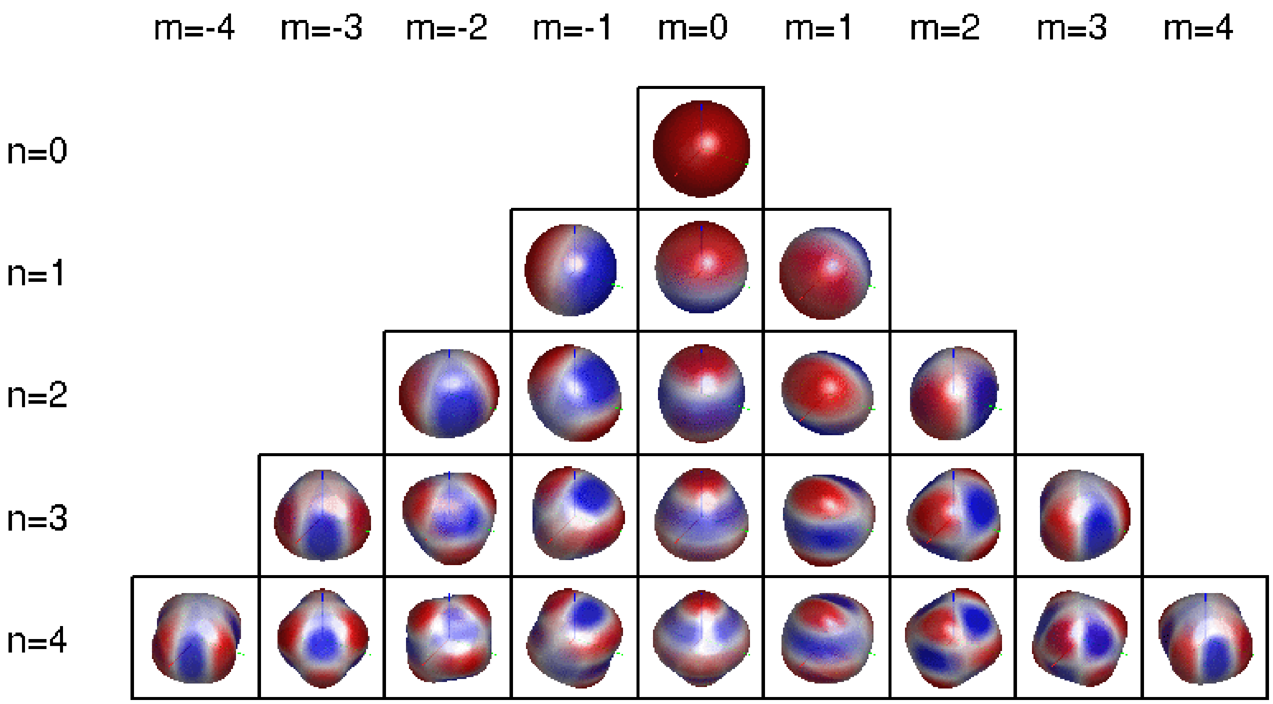
$$p_{\mathrm{N}}(\varphi, \vartheta, \omega) = \sum_{n=0}^{\mathrm{N}} \sum_{m=-n}^{n} Y_n^m(\varphi, \vartheta) \cdot \psi_{nm}(\omega)$$



# Spherical Harmonics

$$Y_n^m(\varphi,\vartheta) = N_n^m P_n^m(\cos(\vartheta)) \begin{cases} \sin(m\varphi), & \text{for } m < 0 \\ \cos(m\varphi), & \text{for } m \geq 0 \end{cases}$$



# Theory of Sound-Radiation Analysis
## Surrounding Spherical Mic-Array

- Sound pressure

Image: put musician and instrument
into a big enough mic-array

**How do we interpolate?**

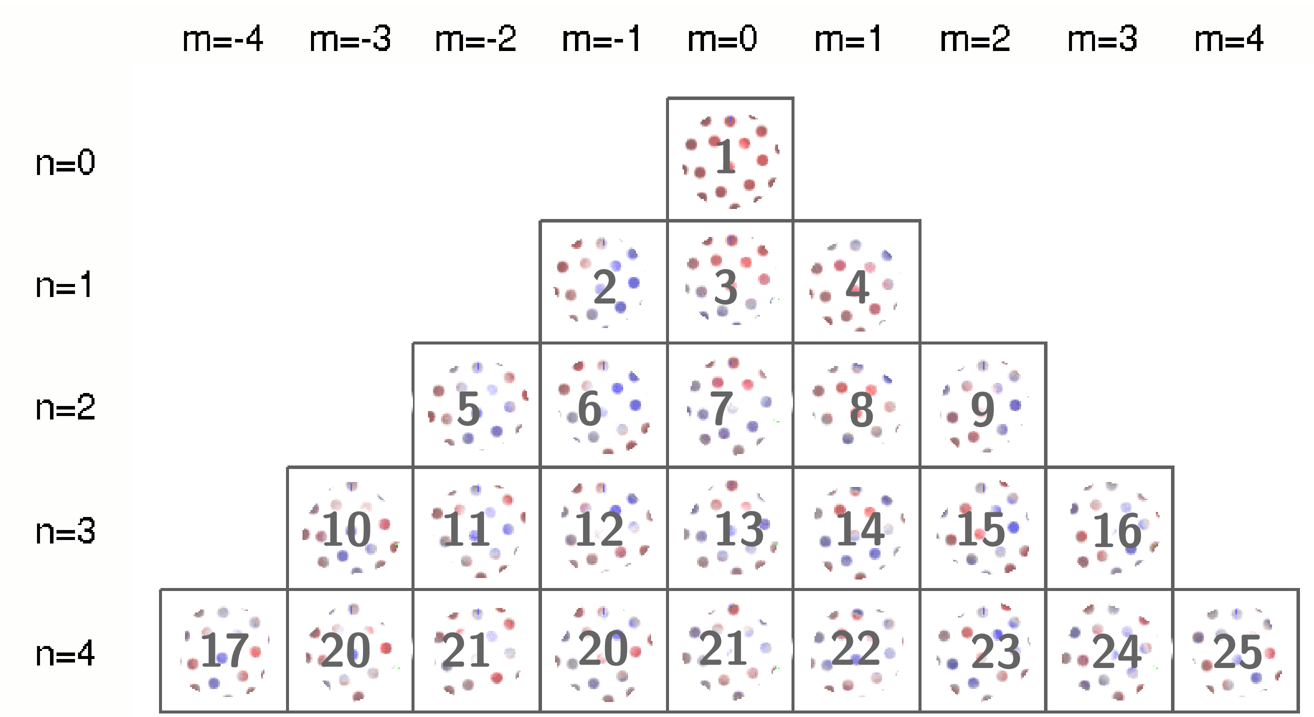
$$\boldsymbol{p}(t) = \begin{bmatrix} p(\varphi_1, \vartheta_1, t) \\ \vdots \\ p(\varphi_\mathrm{K}, \vartheta_\mathrm{K}, t) \end{bmatrix}$$

# Decomposition of Sound-Radiation Signals

radiation pattern with finite angular bandwidth

$$\boldsymbol{p}_{\mathrm{N}}(t) = \sum_{n=0}^{\mathrm{N}} \sum_{m=-n}^{n} \boldsymbol{y}_n^m \cdot \psi_{nm}(t)$$

$$\boldsymbol{y}_n^m = \begin{bmatrix} Y_n^m(\varphi_1, \vartheta_1) \\ \vdots \\ Y_n^m(\varphi_{\mathrm{K}}, \vartheta_{\mathrm{K}}) \end{bmatrix}$$



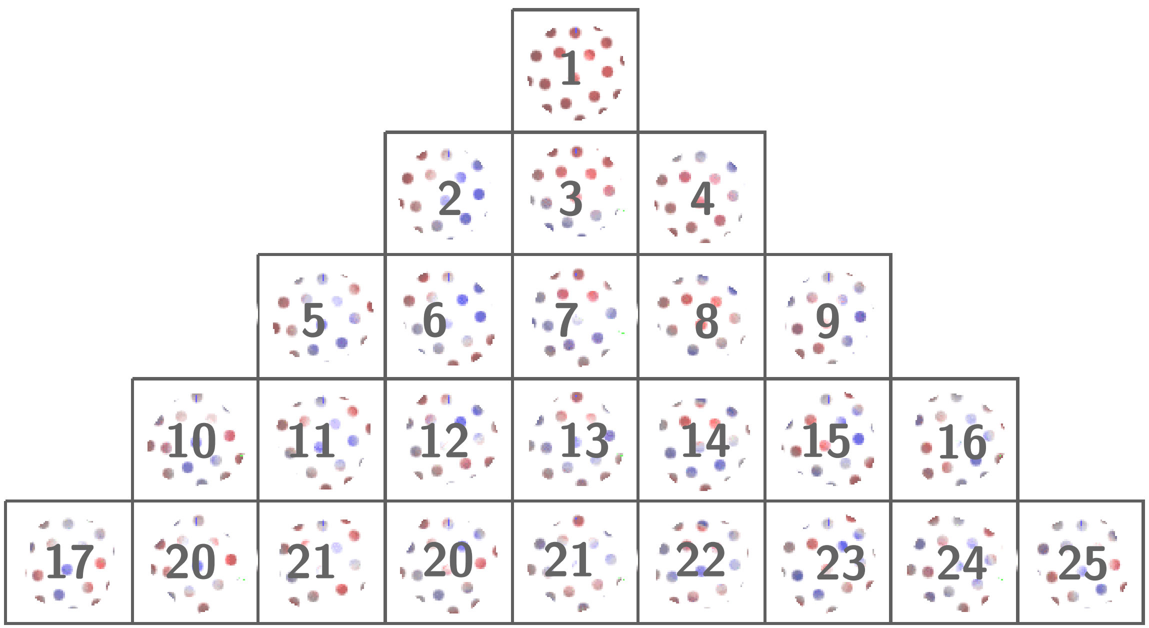
# Decomposition of Sound-Radiation Signals

radiation pattern with finte angular bandwith

$$\boldsymbol{p}_{\mathrm{N}}(t) = \boldsymbol{Y}_{\mathrm{N}} \cdot \boldsymbol{\psi}_{\mathrm{N}}(t)$$

$$\boldsymbol{y}_n^m = \begin{bmatrix} Y_n^m(\varphi_1, \vartheta_1) \\ \vdots \\ Y_n^m(\varphi_{\mathrm{K}}, \vartheta_{\mathrm{K}}) \end{bmatrix}$$

$$\boldsymbol{Y}_{\mathrm{N}} = \left[\boldsymbol{y}_0^0 \cdots \boldsymbol{y}_{-\mathrm{N}}^{\mathrm{N}} \cdots \boldsymbol{y}_{\mathrm{N}}^0 \cdots \boldsymbol{y}_{\mathrm{N}}^{\mathrm{N}}\right]$$




$$= \begin{bmatrix} \psi_0^0(t) \\ \vdots \\ \psi_{\mathrm{N}}^{-\mathrm{N}}(t) \\ \vdots \\ \psi_{\mathrm{N}}^0(t) \\ \vdots \\ \psi_{\mathrm{N}}^{\mathrm{N}}(t) \end{bmatrix}$$

# Decomposition of Sound-Radiation Signals

inversion yields coefficients

$$\boldsymbol{p}_{\mathrm{N}}(t) = \boldsymbol{Y}_{\mathrm{N}} \cdot \boldsymbol{\psi}_{\mathrm{N}}(t)$$

$$\boldsymbol{\psi}_{\mathrm{N}}(t) = \boldsymbol{Y}_{\mathrm{N}}^{\text{"}-1\text{"}} \cdot \boldsymbol{p}(t)$$

$\mathrm{K} \geq (\mathrm{N}+1)^2$

$\mathrm{K} \ldots$ microphones

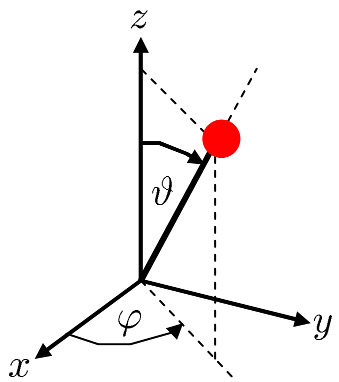
$\mathrm{N} \ldots$ SH order

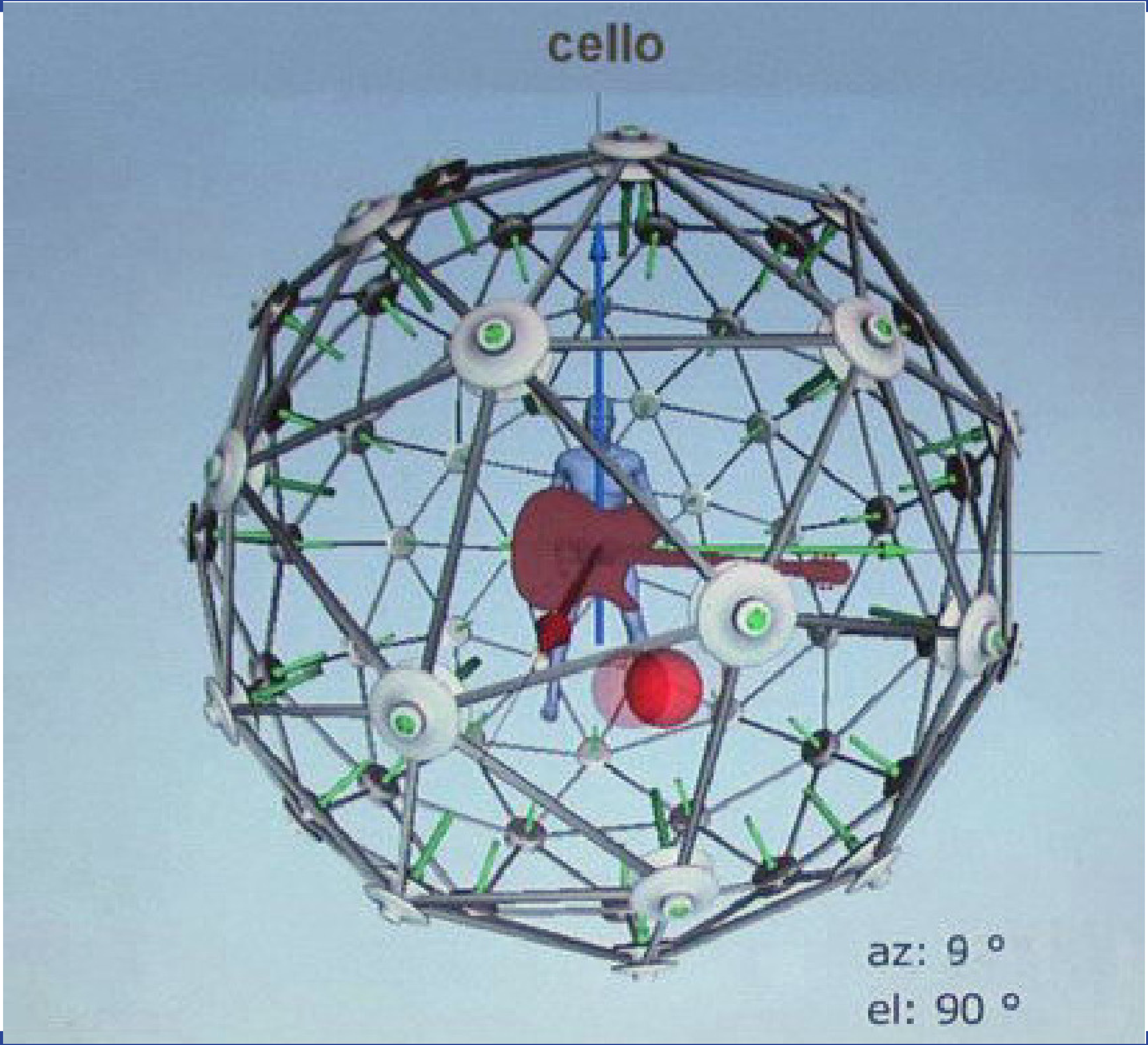
This is how we interpolate (DEMO):

$$p_{\mathrm{N}}(\varphi,\vartheta,t) = \left[Y_0^0(\varphi,\vartheta) \cdots Y_{-\mathrm{N}}^{\mathrm{N}}(\varphi,\vartheta) \cdots Y_{\mathrm{N}}^{0}(\varphi,\vartheta) \cdots Y_{\mathrm{N}}^{\mathrm{N}}(\varphi,\vartheta)\right] \cdot \boldsymbol{\psi}_{\mathrm{N}}(t)$$

Nachbar, Nistelberger 2009.

Baumgartner, Messner 2010.

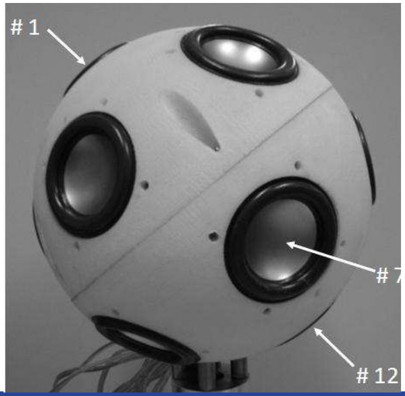



cello
az: 9 °
el: 90 °

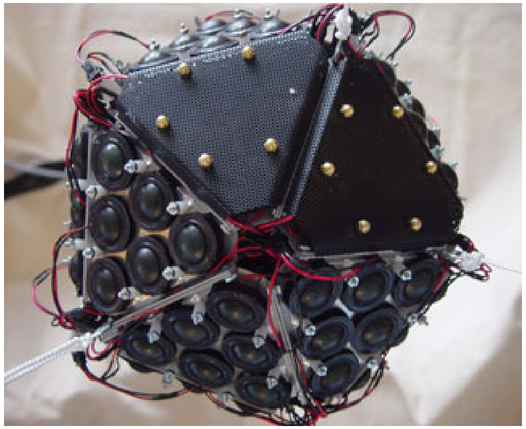
# Compact Spherical Loudspeaker Arrays Suitable for Holophony of Sound Radiation

Warusfel, Caussé, et al 1997 (FR)

Pasqual, Herzog, Arruda, 2008-2010 (BR/FR)




Avizienis, Freed, Kassakian, Wessel, 2006 Schmeder 2009 (US)



Behler, Witew, Pollow, 2006-2010 (DE)

Rafaely, Peleg, 2009-2010 (IL)

Zotter, Pomberger, Kerscher, 2007-2010 (AT)

# Measuring the Directivity of a Compact Spherical Loudspeaker Array

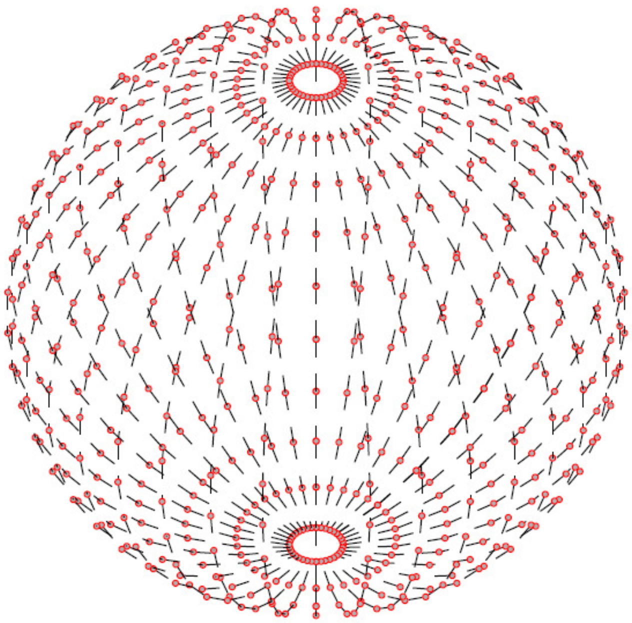


**Figure 1:** Sampling Layout

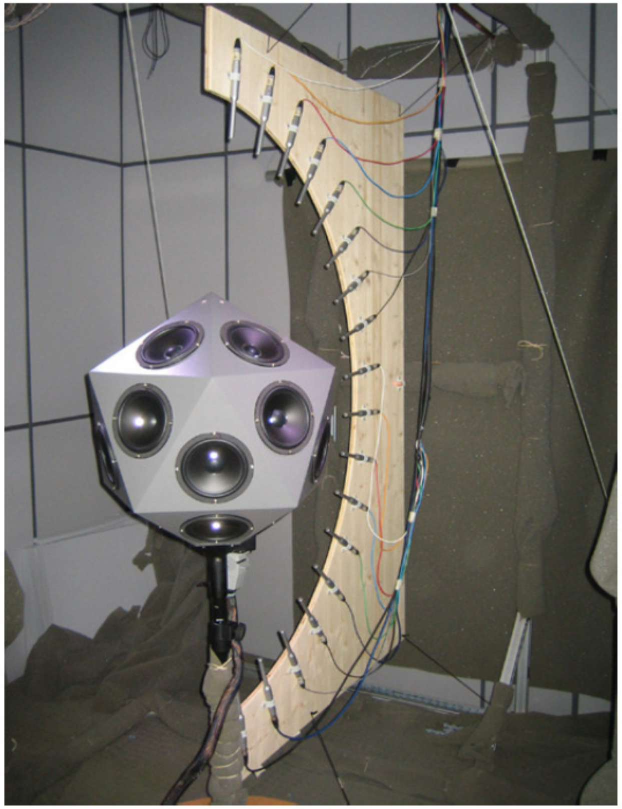
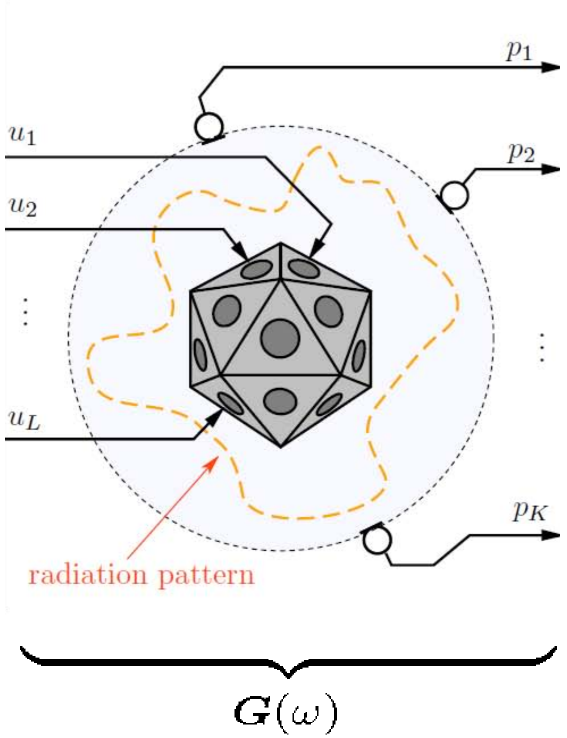


**Figure 2:** Icosahedron and Semicircular Microphone Array

# Directivity of a Compact Spherical Loudspeaker Array: An Acoustic MIMO-System

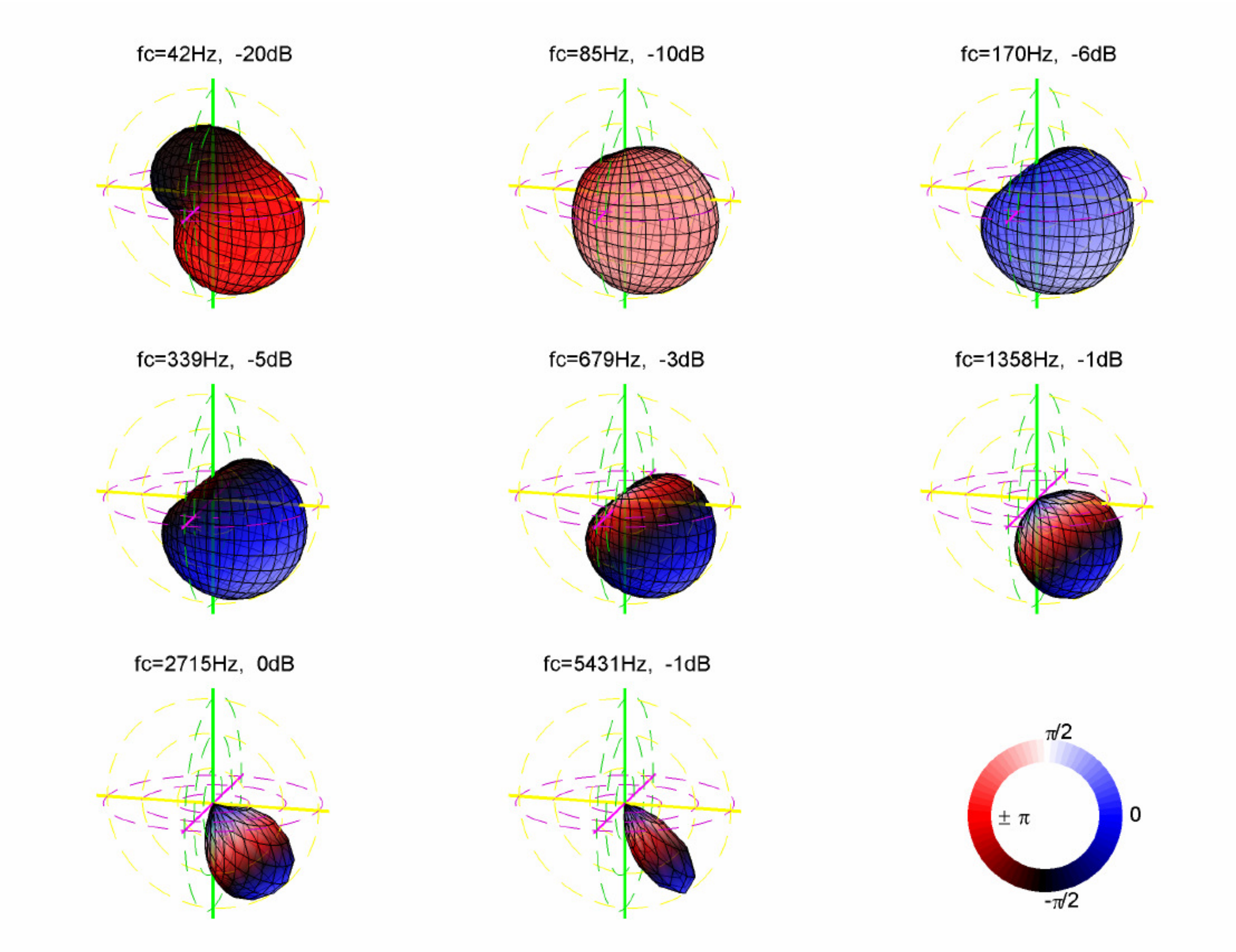



$\mathrm{K} > \mathrm{L}$ ... microphones

$\mathrm{L}$ ... loudspeakers

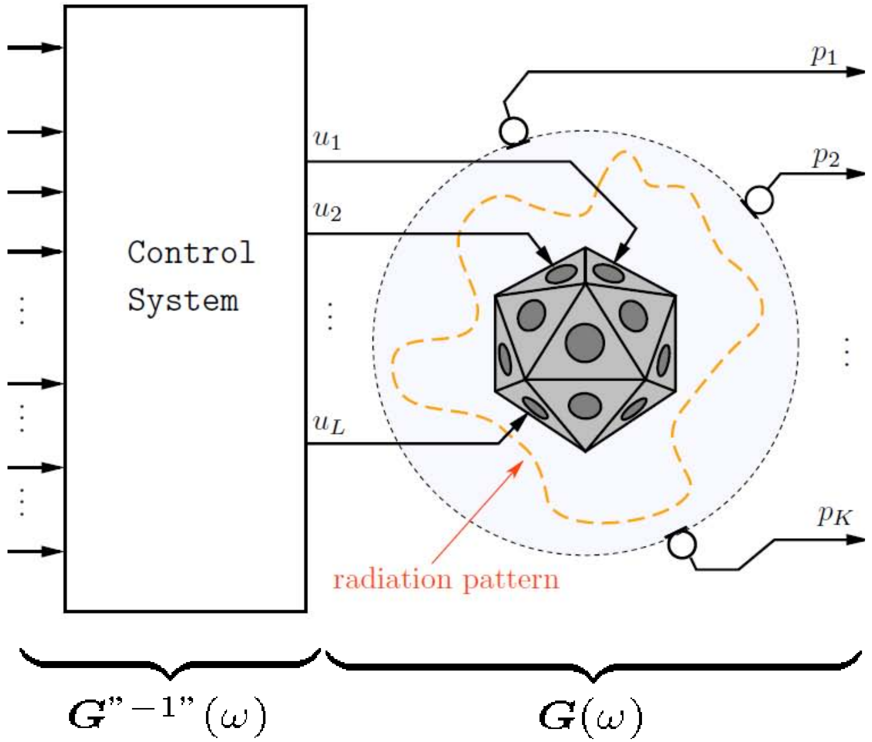
$$\begin{bmatrix} p_1(\omega) \\ \vdots \\ p_{\mathrm{K}}(\omega) \end{bmatrix} = \begin{bmatrix} G_{11}(\omega) & \cdots & G_{1\mathrm{L}}(\omega) \\ \vdots & \ddots & \vdots \\ G_{\mathrm{K}1}(\omega) & \cdots & G_{\mathrm{KL}}(\omega) \end{bmatrix} \cdot \begin{bmatrix} u_1(\omega) \\ \vdots \\ u_{\mathrm{L}}(\omega) \end{bmatrix}$$

# Directivity of One Array-Speaker




$$\boldsymbol{p}_1(\omega) = \boldsymbol{G}(\omega) \cdot \begin{bmatrix} 1 \\ 0 \\ \vdots \end{bmatrix}$$

# Directivity of a Compact Spherical Loudspeaker Array: An Acoustic MIMO-System

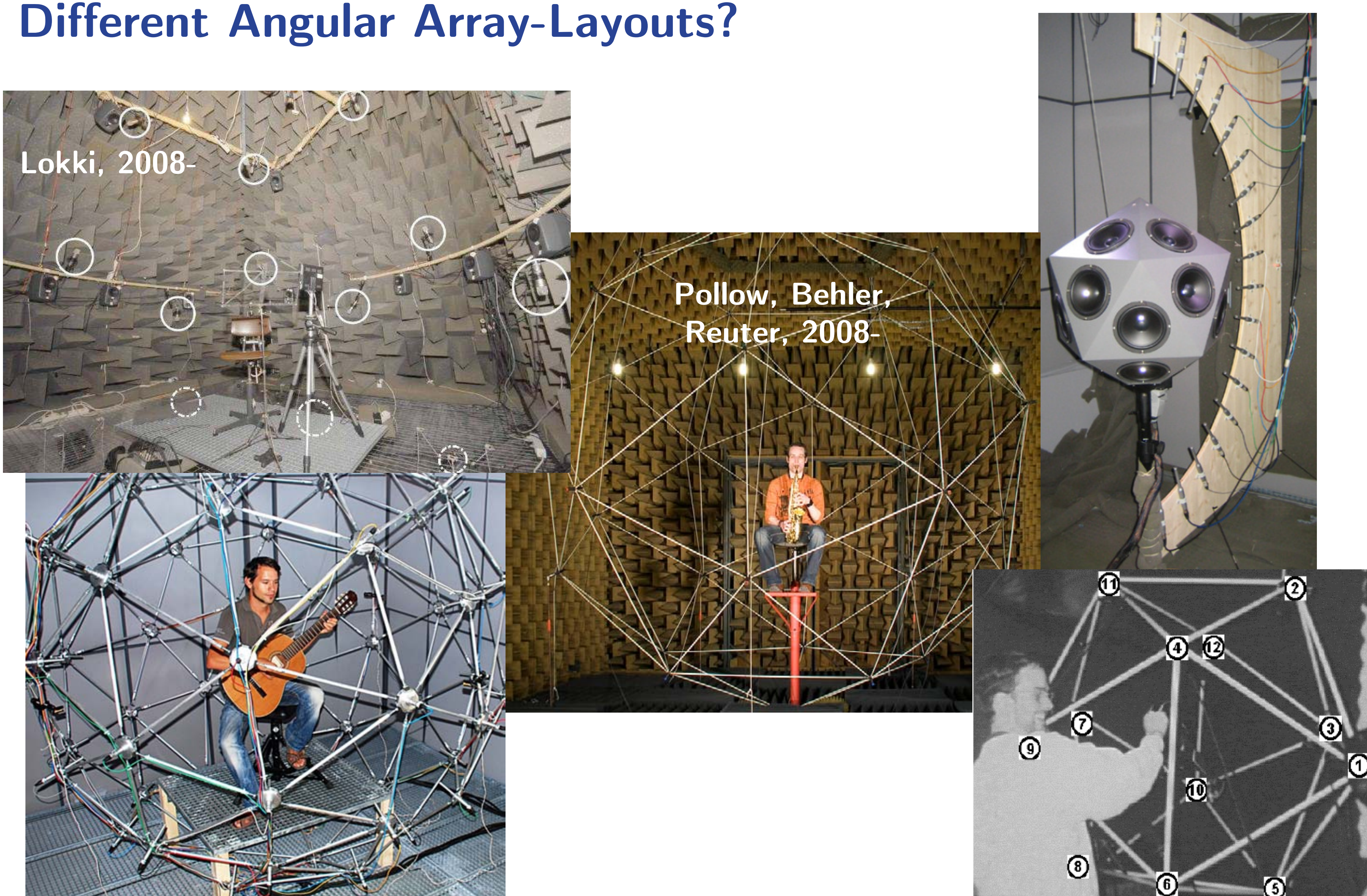



$$\boldsymbol{p}(\omega) \approx \hat{\boldsymbol{p}}(\omega)$$

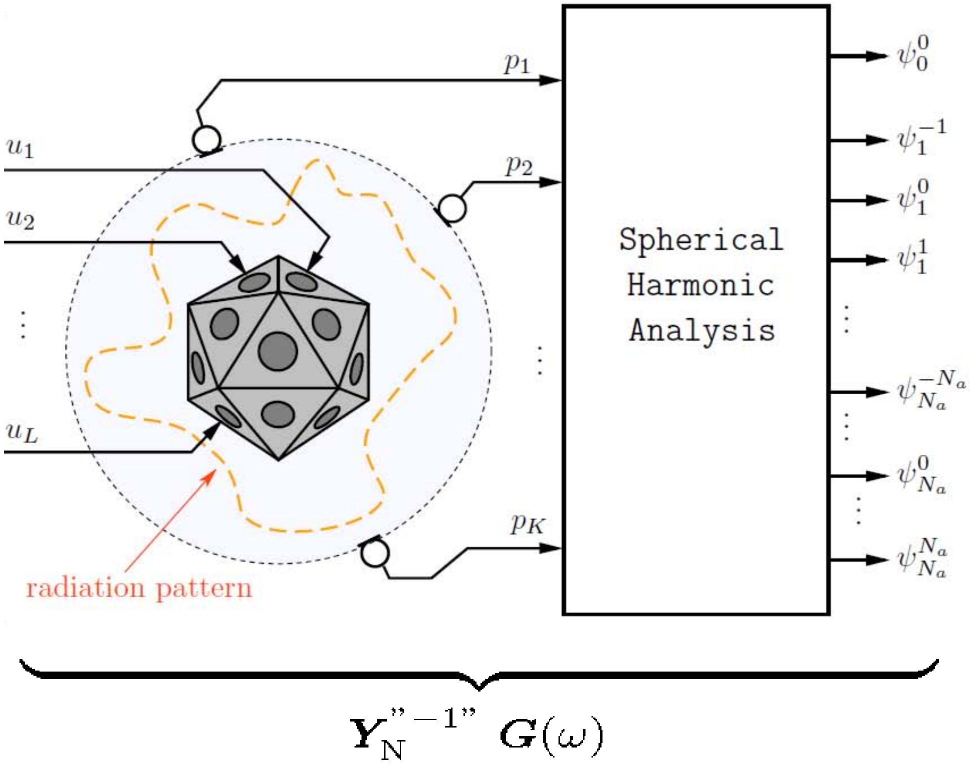
$$\boldsymbol{p}(\omega) = \boldsymbol{G}(\omega) \cdot \boldsymbol{u}(\omega)$$

$$\boldsymbol{u}(\omega) = \boldsymbol{G}^{\text{"}-1\text{"}}(\omega) \cdot \hat{\boldsymbol{p}}(\omega)$$

# Different Angular Array-Layouts?

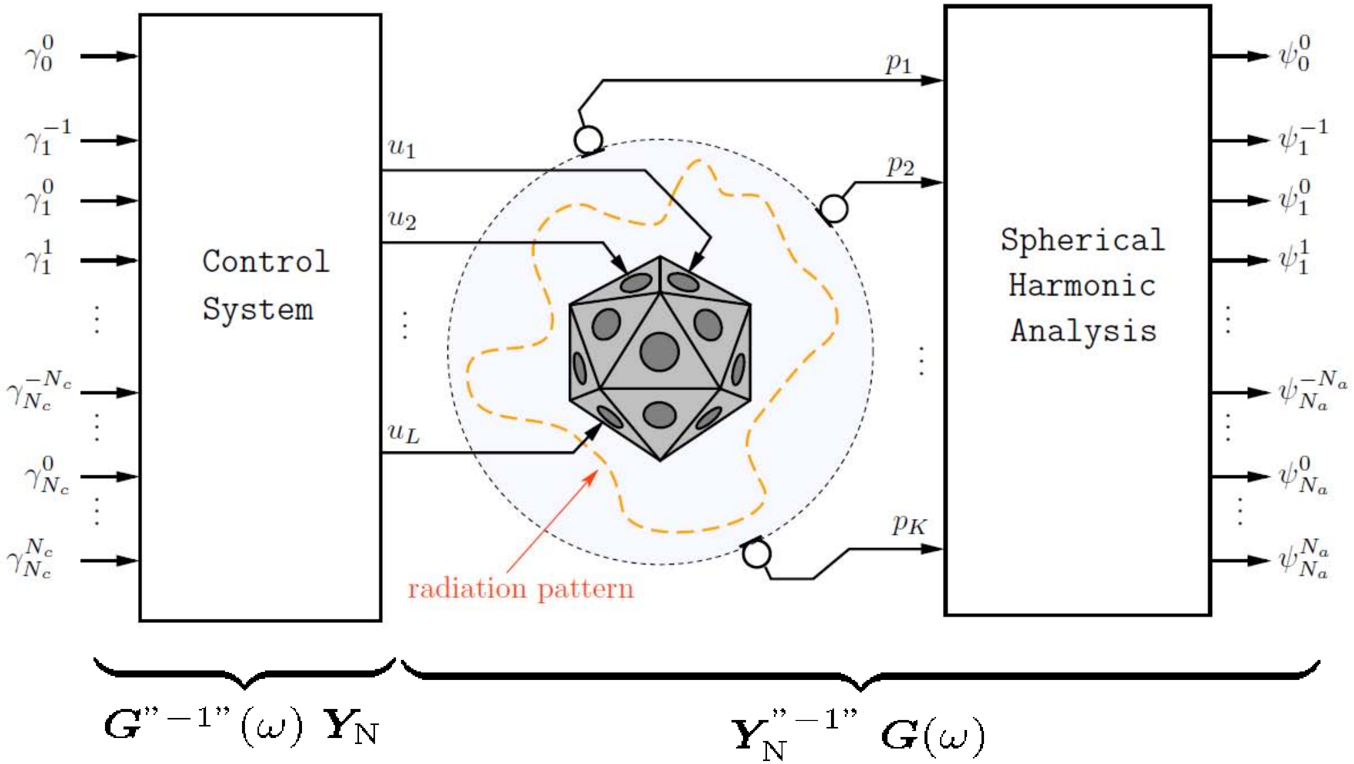


# Directivity of a Compact Spherical Loudspeaker Array: An Acoustic MIMO-System




$$\boldsymbol{\psi}_N(\omega) = \boldsymbol{Y}_N^{"-1"}\ \boldsymbol{G}(\omega) \cdot \boldsymbol{u}(\omega)$$

# Directivity of a Compact Spherical Loudspeaker Array: An Acoustic MIMO-System

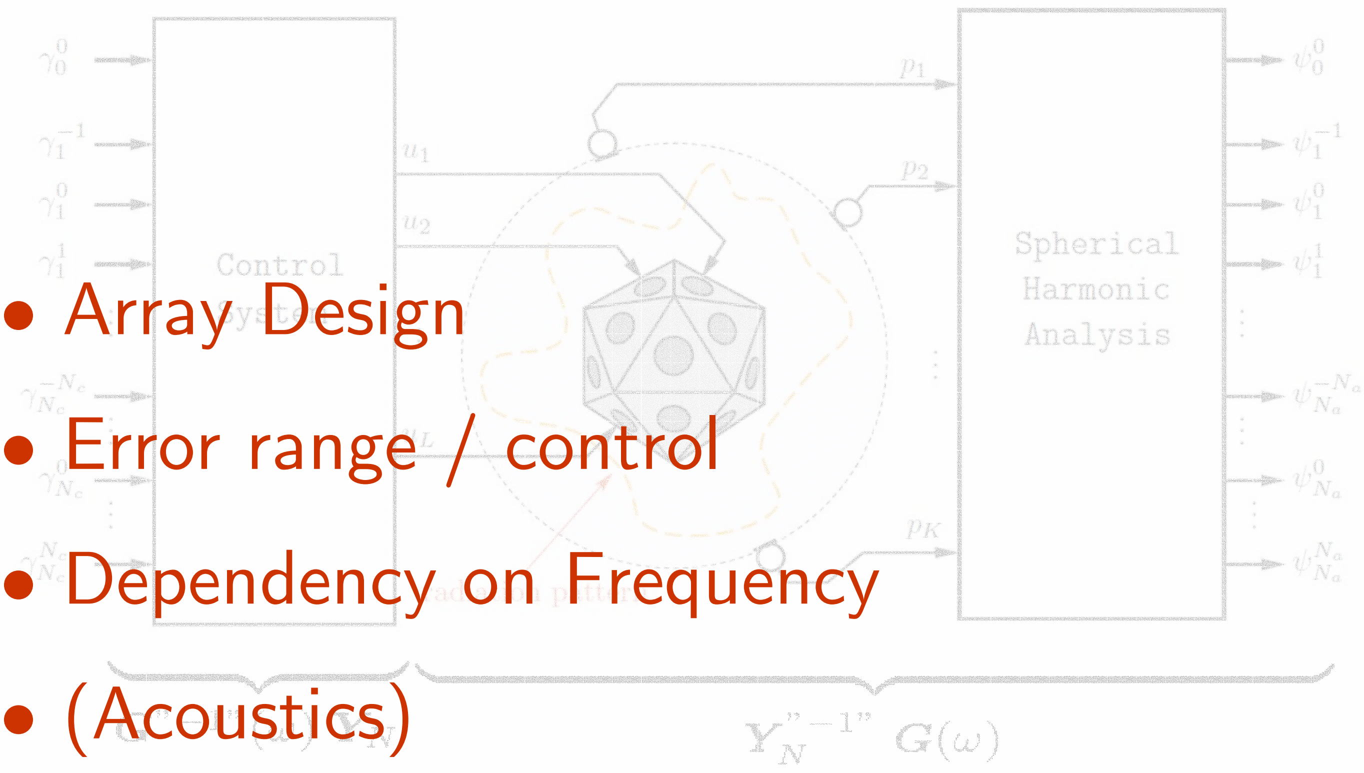



$$\boldsymbol{\psi}_{\mathrm{N}}(\omega) \approx \boldsymbol{\gamma}_{\mathrm{N}}(\omega)$$

$$\boldsymbol{\psi}_{\mathrm{N}}(\omega) = \boldsymbol{Y}_{\mathrm{N}}^{"-1"}\ \boldsymbol{G}(\omega) \cdot \boldsymbol{u}(\omega)$$

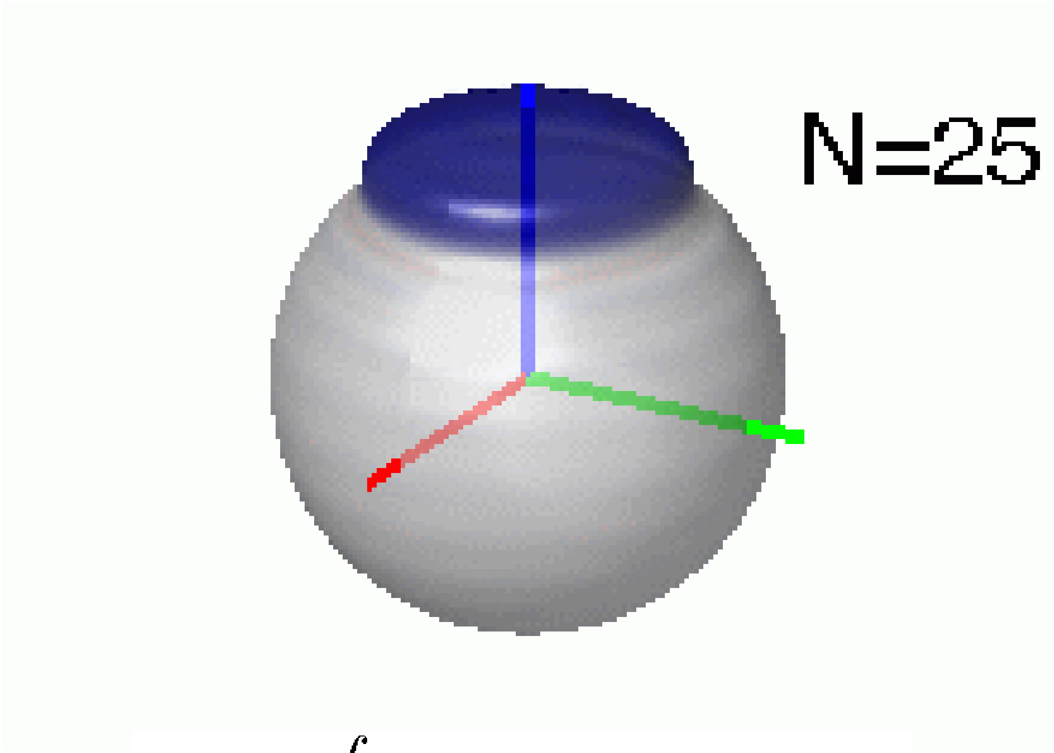
$$\boldsymbol{u}(\omega) = \boldsymbol{G}^{"-1"}(\omega)\ \boldsymbol{Y}_{\mathrm{N}} \cdot \boldsymbol{\gamma}_{\mathrm{N}}(\omega)$$

# Directivity of a Compact Spherical Loudspeaker Array: An Acoustic MIMO-System




- Array Design
- Error range / control
- Dependency on Frequency
- (Acoustics)

# Compact Spherical Loudspeaker Array Surface $r_0$

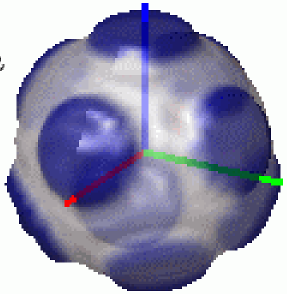



$$\nu_{nm} = \int_{\mathbb{S}^2} v(\varphi,\vartheta)\, Y_n^m(\varphi,\vartheta)\, dS$$

$$v(\varphi,\vartheta) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} Y_n^m(\varphi,\vartheta) \cdot \nu_{nm}$$

# Compact Spherical Loudspeaker Array Surface $r_0$

$$v(\varphi,\vartheta)=\sum_{n=0}^{\infty}\sum_{m=-n}^{n}Y_n^m(\varphi,\vartheta)\cdot\nu_{nm}$$



N=25

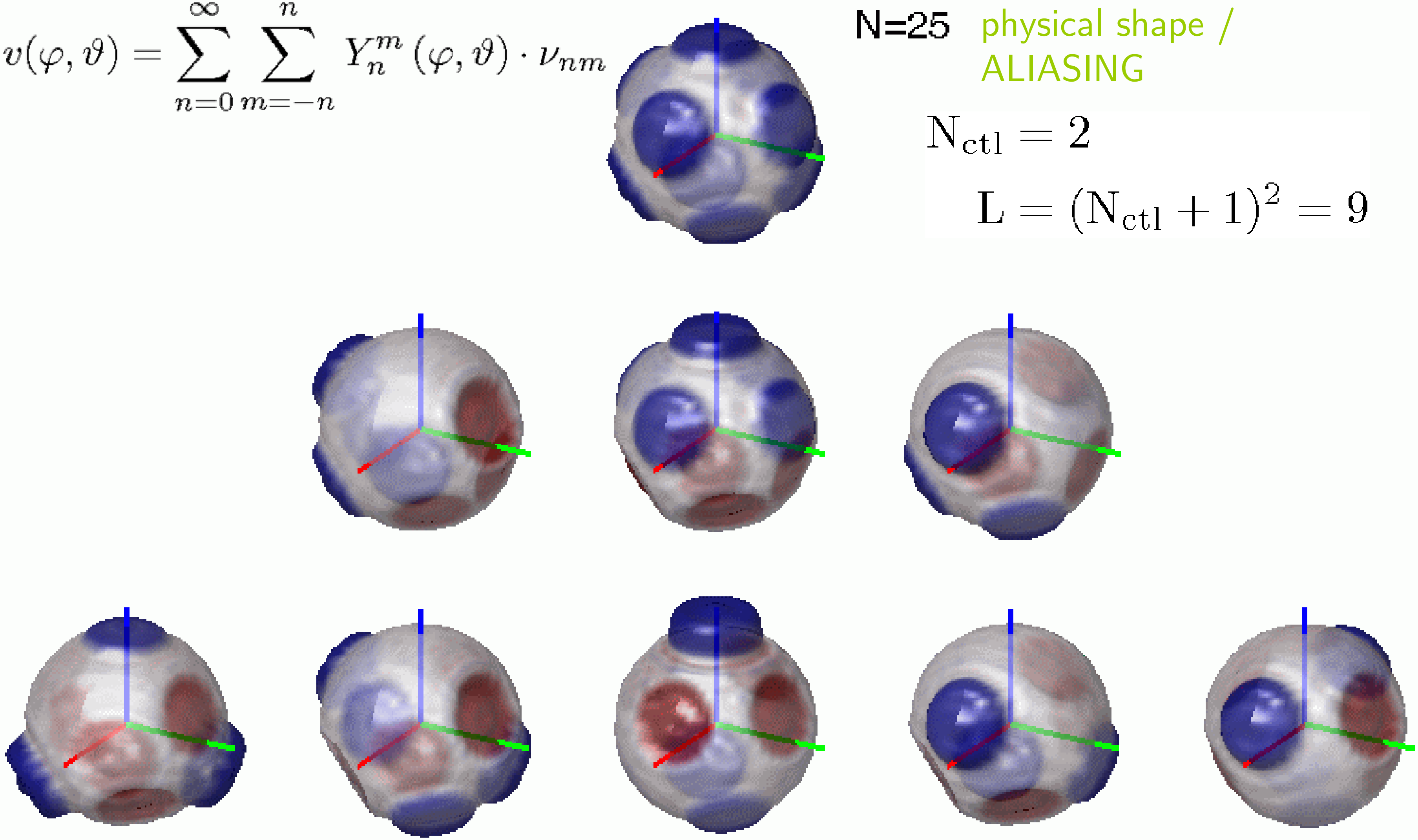
# Compact Spherical Loudspeaker Array Surface $r_0$

$$v(\varphi,\vartheta)=\sum_{n=0}^{\infty}\sum_{m=-n}^{n}Y_n^m(\varphi,\vartheta)\cdot\nu_{nm}$$

N=25 physical shape / ALIASING

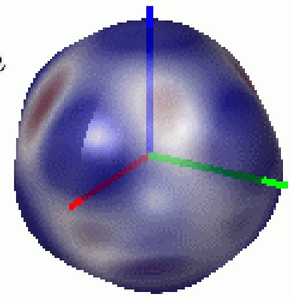
$$\mathrm{N}_{\mathrm{ctl}}=2$$

$$\mathrm{L}=(\mathrm{N}_{\mathrm{ctl}}+1)^2=9$$



# Compact Spherical Loudspeaker Array Surface $r_0$

$$v_5(\varphi,\vartheta) = \sum_{n=0}^{5} \sum_{m=-n}^{n} Y_n^m(\varphi,\vartheta) \cdot \nu_{nm}$$

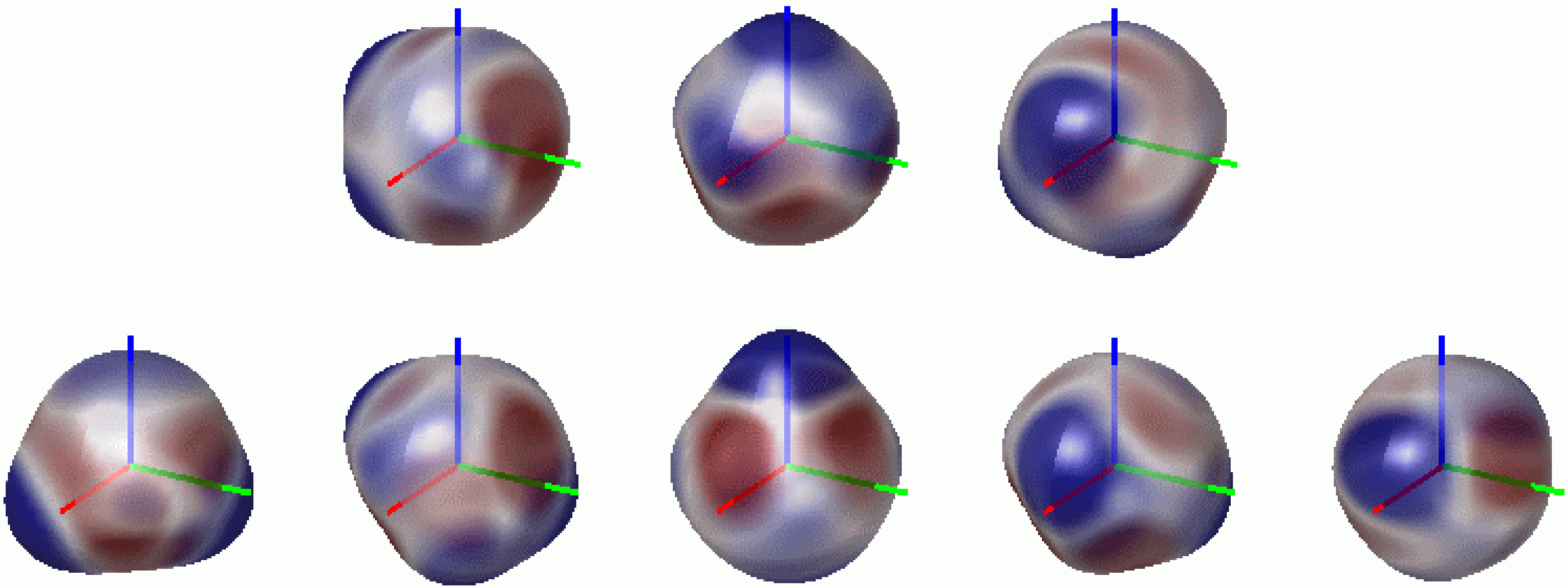


N=5

unphysical attempt:
ALIASING SUPPRESSION

$$\mathrm{N}_{\mathrm{ctl}} = 2$$

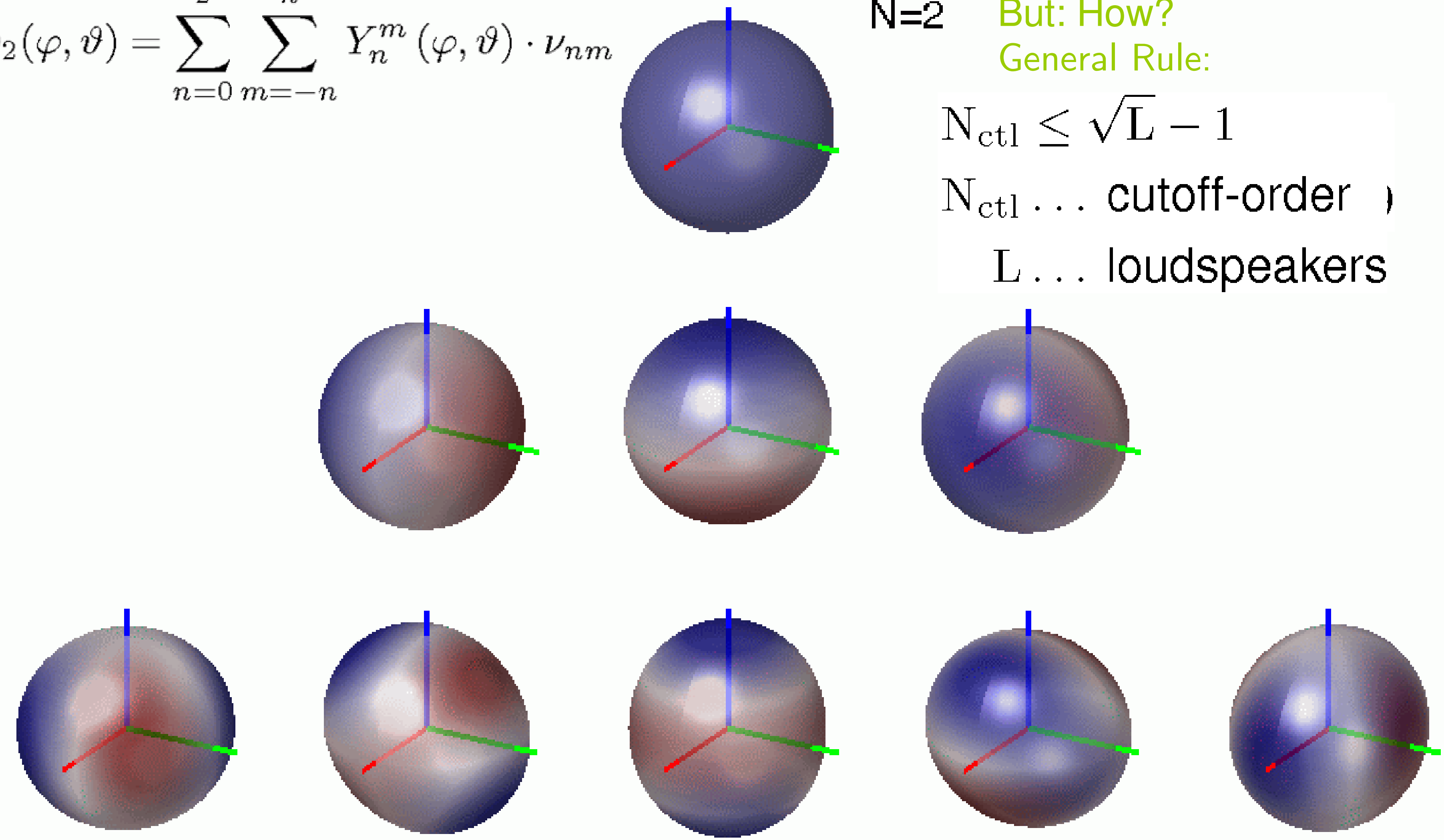
$$\mathrm{L} = (\mathrm{N}_{\mathrm{ctl}} + 1)^2 = 9$$



# Compact Spherical Loudspeaker Array Surface $r_0$

$$v_2(\varphi,\vartheta)=\sum_{n=0}^{2}\sum_{m=-n}^{n} Y_n^m(\varphi,\vartheta)\cdot\nu_{nm}$$

N=2



But: How?
General Rule:

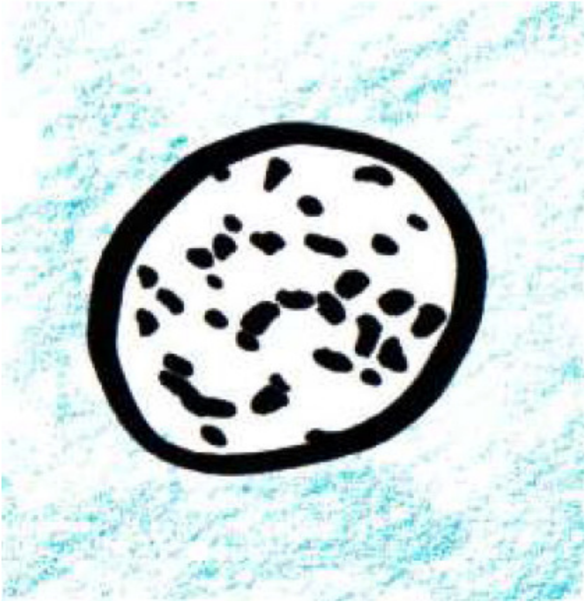
$$\mathrm{N_{ctl}} \leq \sqrt{\mathrm{L}}-1$$

$\mathrm{N_{ctl}}$ ... cutoff-order

L ... loudspeakers

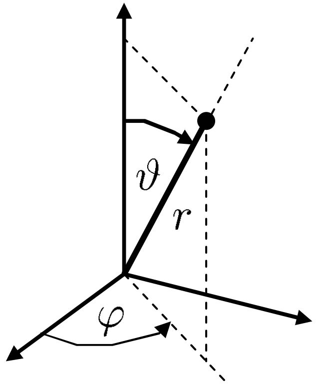
# Compact Spherical Loudspeaker Array Radiation

- Spherical Boundary Value Problem (Neumann)



Helmholtz-equation (ac.wv.)

$$\left(k^2 + \triangle\right) p = 0$$




Laplace-operator (spherical coordinates, chain rule)

$$\triangle_{r,\varphi,\vartheta}\, p = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial p}{\partial r} \right) + \frac{1}{r^2 \sin(\vartheta)} \frac{\partial}{\partial \vartheta} \left( \sin(\vartheta) \frac{\partial p}{\partial \vartheta} \right) + \frac{1}{r^2 \sin(\vartheta)} \frac{\partial^2 p}{\partial \varphi}$$

$$v(\varphi,\vartheta)\big|_{r_0} = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} Y_n^m(\varphi,\vartheta)\cdot \nu_{nm}\big|_{r_0}$$

$$p(kr,\varphi,\vartheta) = -\mathrm{i}\,\rho_0 c \sum_{n=0}^{\infty}\sum_{m=-n}^{n} Y_n^m(\varphi,\vartheta)\,\frac{h_n(kr)}{h_n'(kr_0)}\,\nu_{nm}\big|_{r_0}$$

small array / long wave (lo f)

$$\frac{r_0}{\lambda} = \frac{1}{4}$$

large array / short wave (hi f)

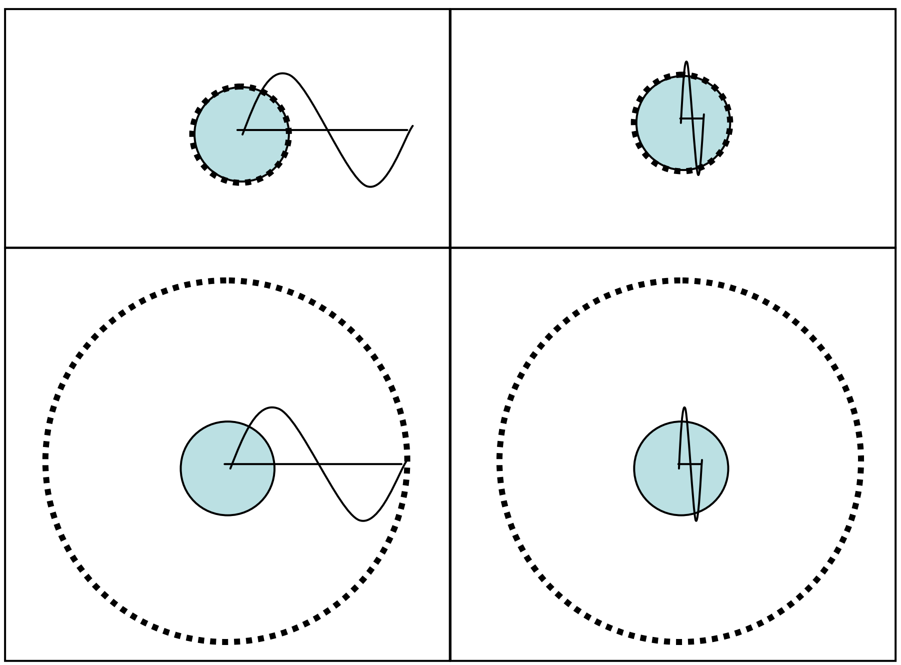
$$\frac{r_0}{\lambda} = 4$$

on surface

$$\frac{r}{r_0} = 1$$

in air

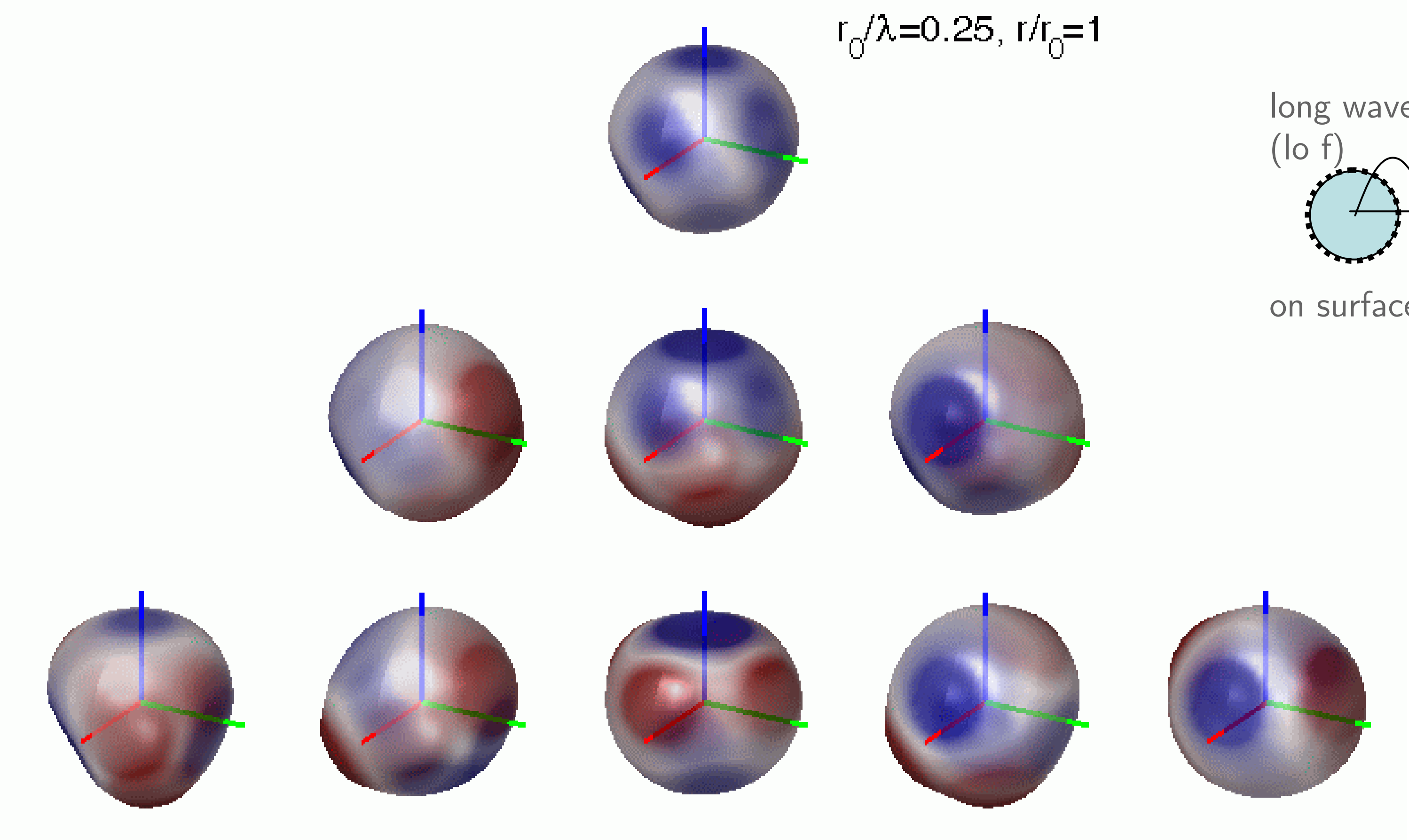
$$\frac{r}{r_0} = 4$$



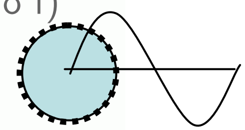
$$kr_0 = 2\pi\frac{r_0}{\lambda}$$

$$v_{\mathrm{N}}(\varphi,\vartheta)=\sum_{n=0}^{N}\sum_{m=-n}^{n} Y_n^m(\varphi,\vartheta)\cdot\nu_{nm}$$

$$p(kr,\varphi,\vartheta)=-\mathrm{i}\,\rho_0 c\sum_{n=0}^{\infty}\sum_{m=-n}^{n} Y_n^m(\varphi,\vartheta)\,\frac{h_n(kr)}{h_n'(kr_0)}\,\nu_{nm}\Big|_{r_0}$$



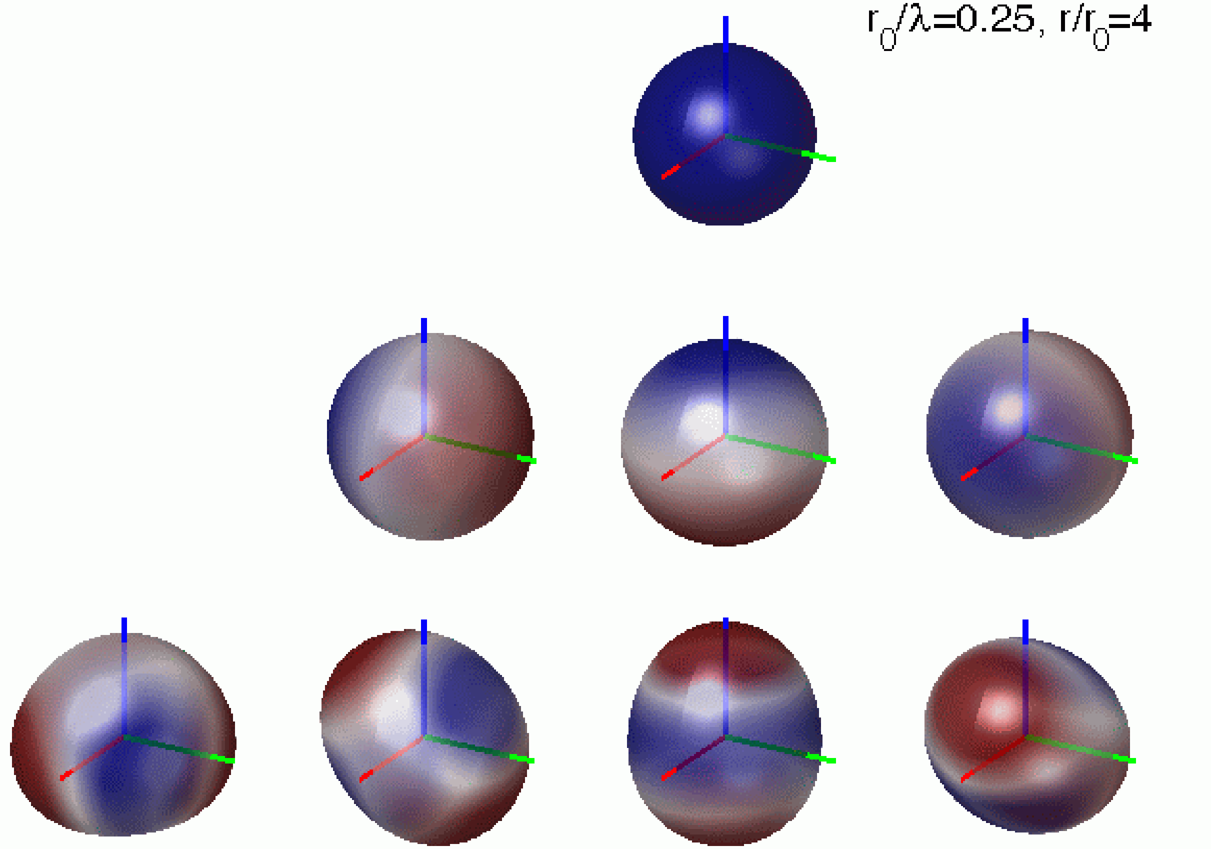

long wave
(lo f)

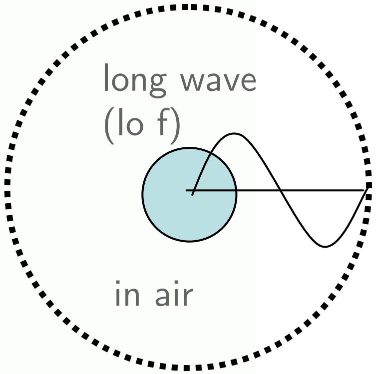


on surface

$$v_{\mathrm{N}}(\varphi,\vartheta) = \sum_{n=0}^{N} \sum_{m=-n}^{n} Y_n^m(\varphi,\vartheta) \cdot \nu_{nm}$$

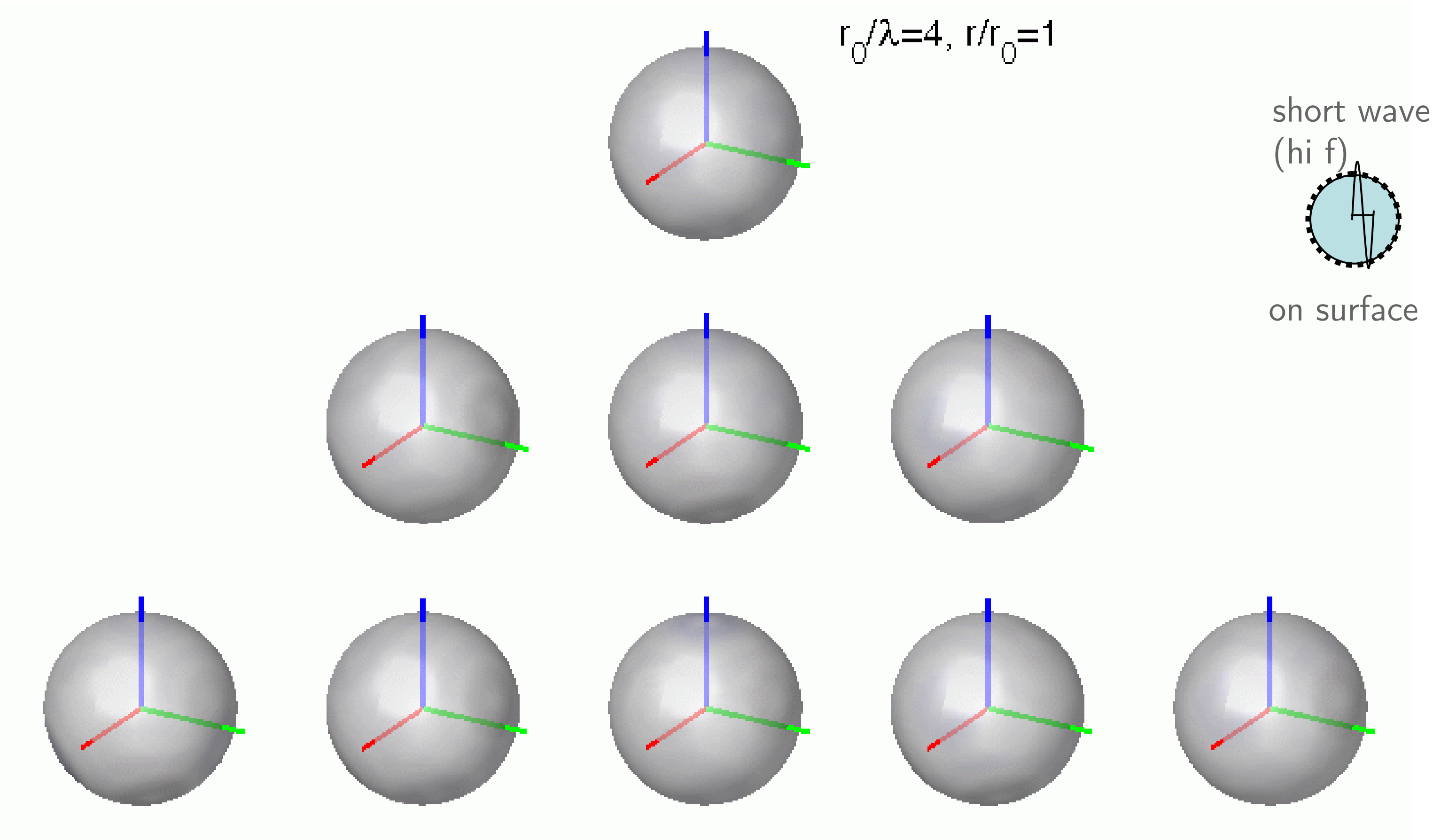
$$p(kr,\varphi,\vartheta) = -\mathrm{i}\,\rho_0 c \sum_{n=0}^{\infty} \sum_{m=-n}^{n} Y_n^m(\varphi,\vartheta) \frac{h_n(kr)}{h_n'(kr_0)} \nu_{nm}\Big|_{r_0}$$



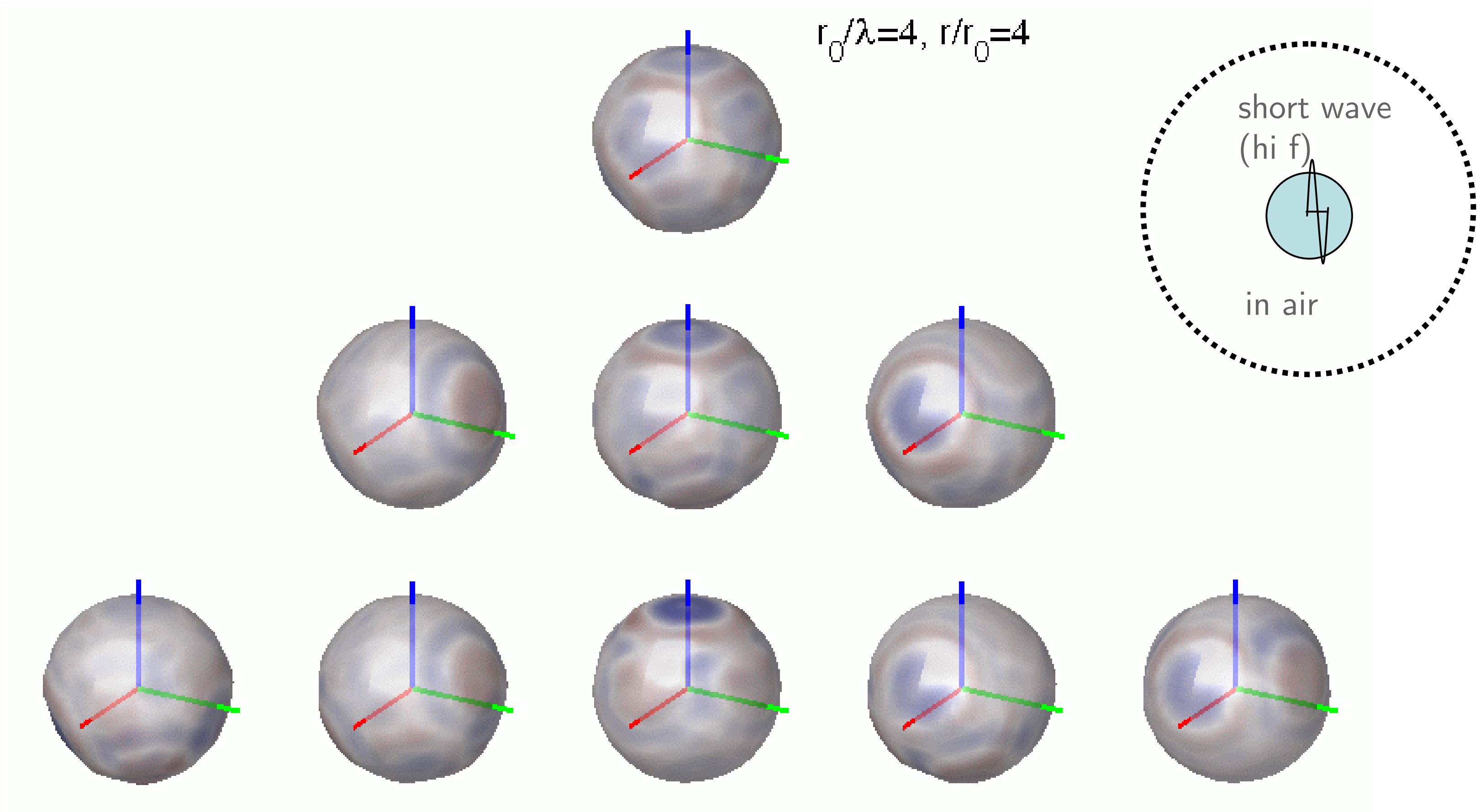
$$v_{\mathrm{N}}(\varphi,\vartheta) = \sum_{n=0}^{N}\sum_{m=-n}^{n} Y_n^m(\varphi,\vartheta)\cdot \nu_{nm}$$

$$p(kr,\varphi,\vartheta) = -\mathrm{i}\,\rho_0 c \sum_{n=0}^{\infty}\sum_{m=-n}^{n} Y_n^m(\varphi,\vartheta)\,\boxed{\frac{h_n(kr)}{h_n'(kr_0)}}\,\nu_{nm}\Big|_{r_0}$$

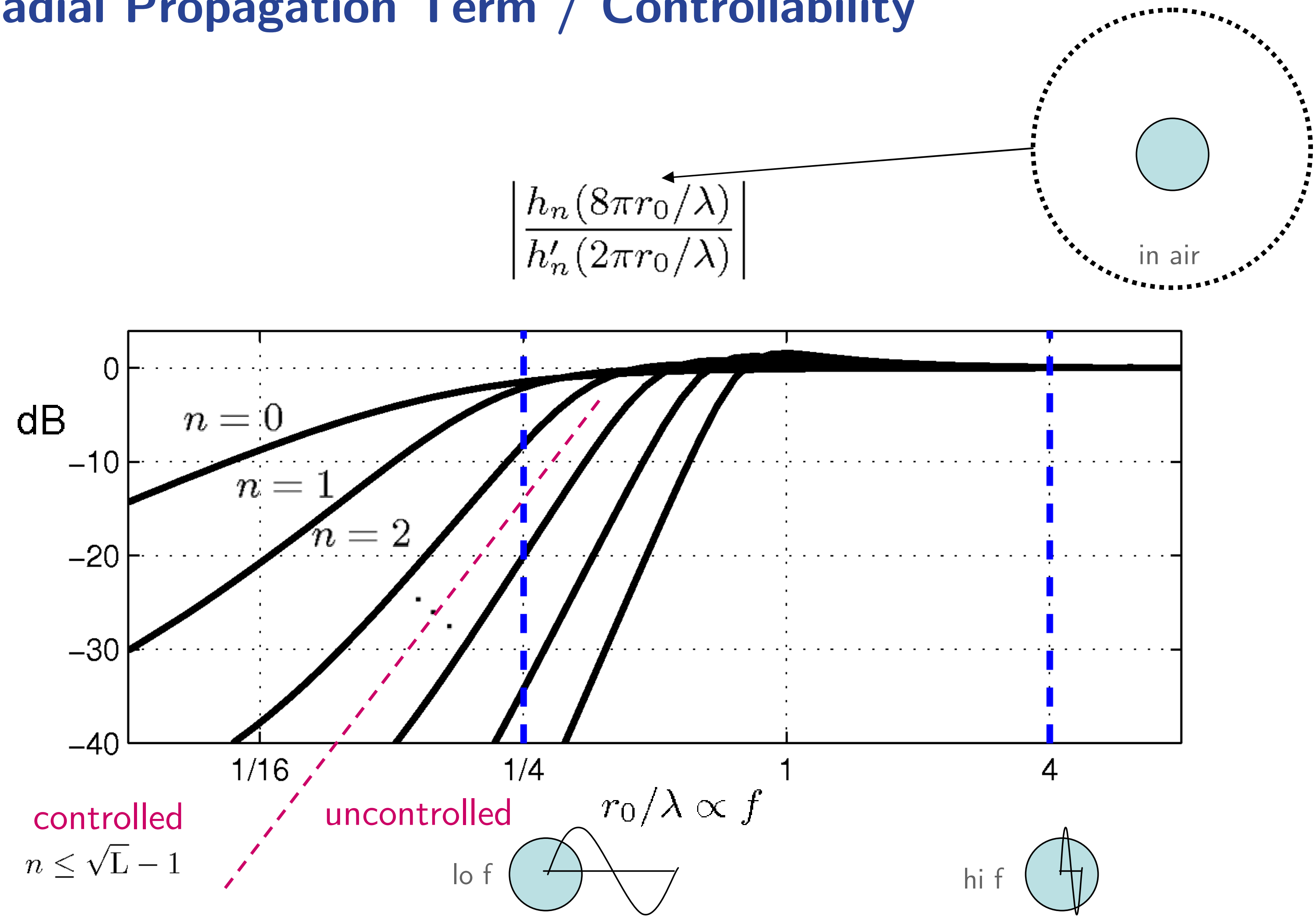


$$v_{\mathrm{N}}(\varphi, \vartheta) = \sum_{n=0}^{N} \sum_{m=-n}^{n} Y_n^m(\varphi, \vartheta) \cdot \nu_{nm}$$

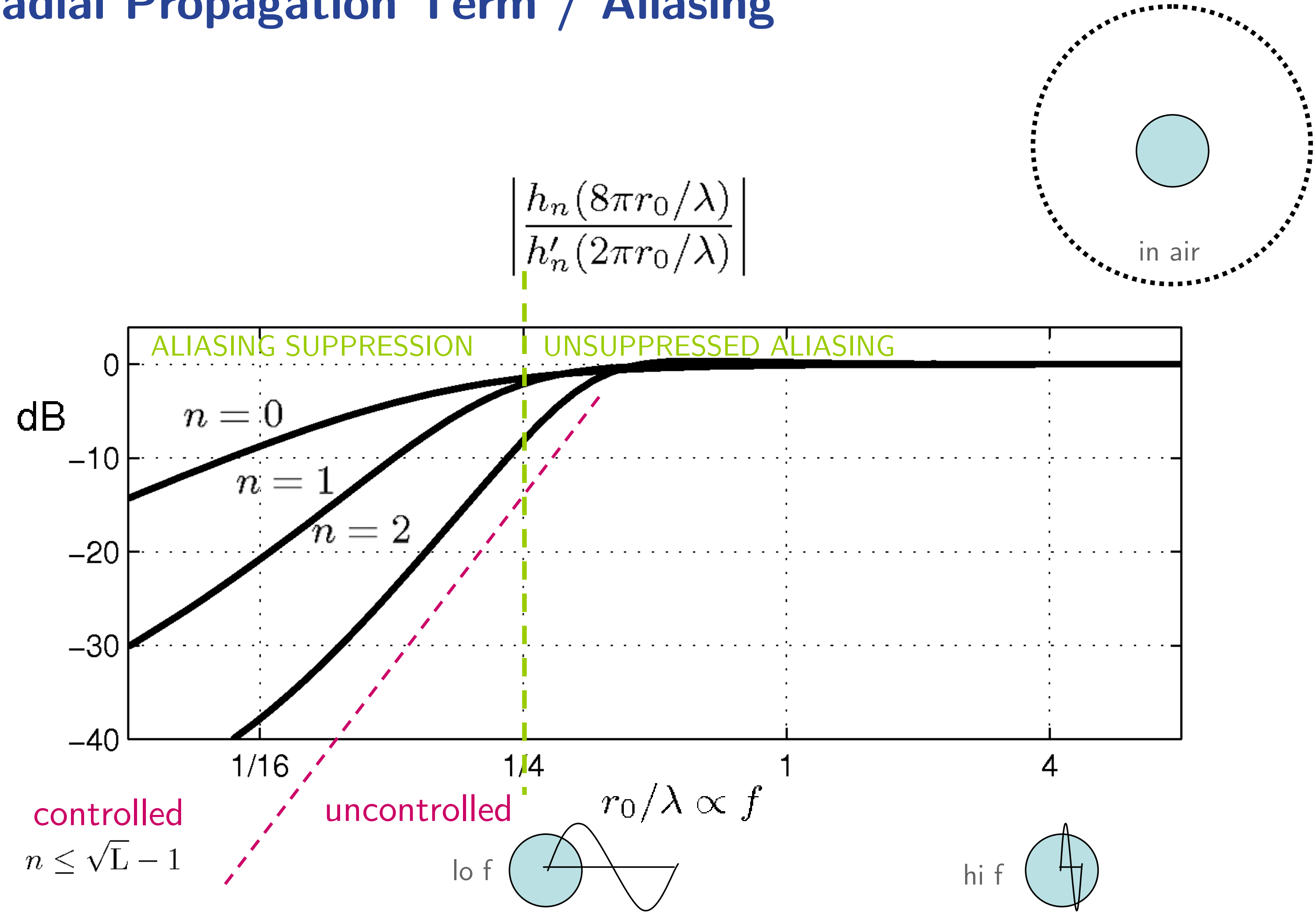
$$p(kr, \varphi, \vartheta) = -\mathrm{i}\,\rho_0 c \sum_{n=0}^{\infty} \sum_{m=-n}^{n} Y_n^m(\varphi, \vartheta) \frac{h_n(kr)}{h_n'(kr_0)} \nu_{nm}\Big|_{r_0}$$



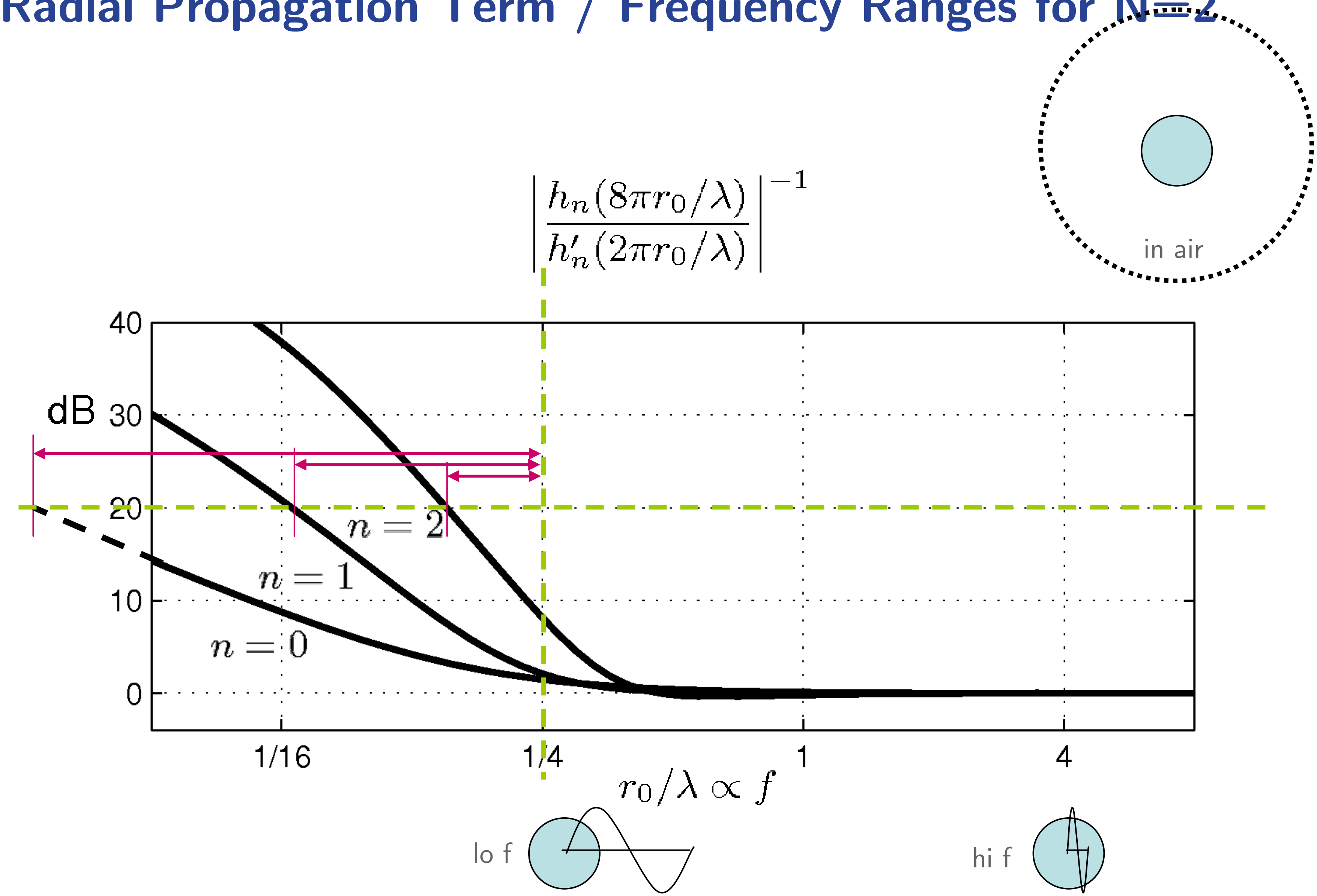
# Radial Propagation Term / Controllability



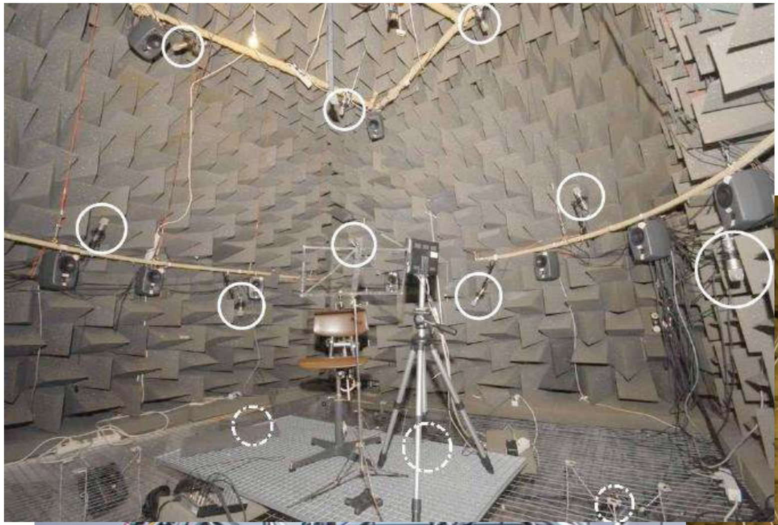
# Radial Propagation Term / Aliasing

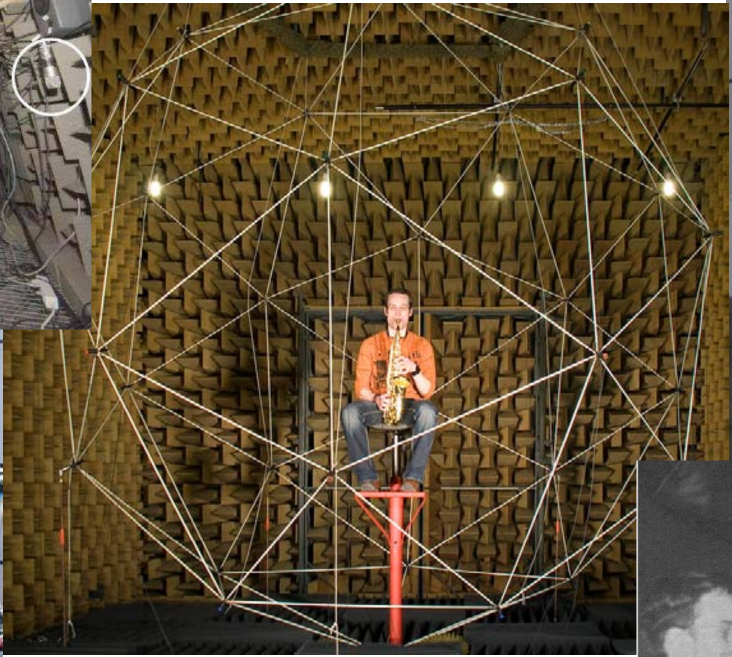


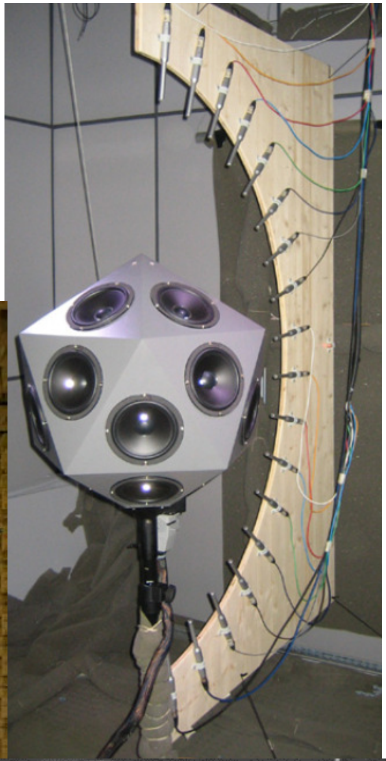
# Radial Propagation Term / Frequency Ranges for N=2

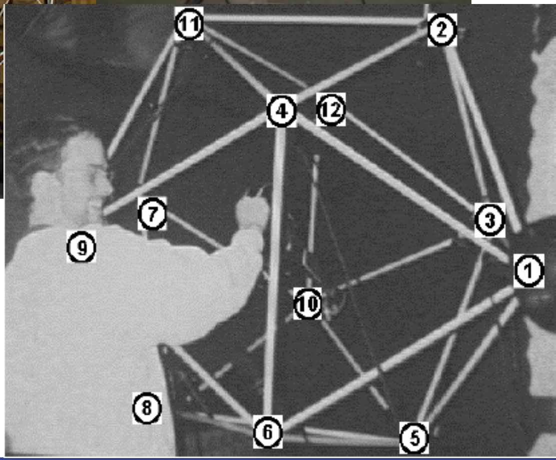


# Different Array-Radii ☺

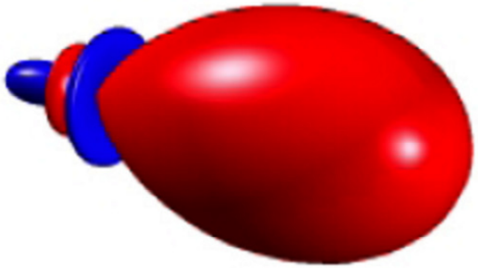


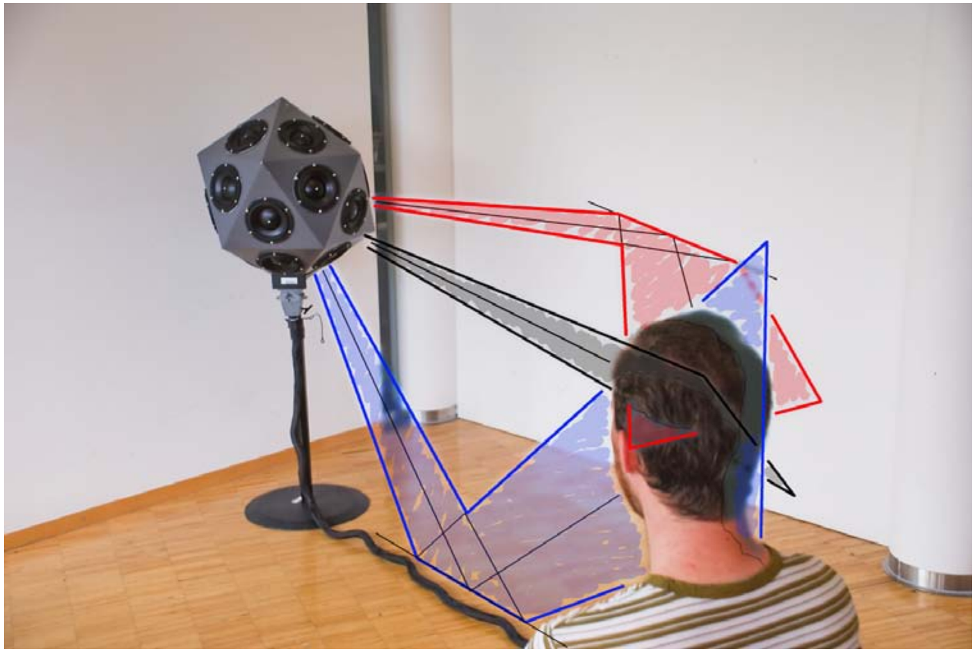




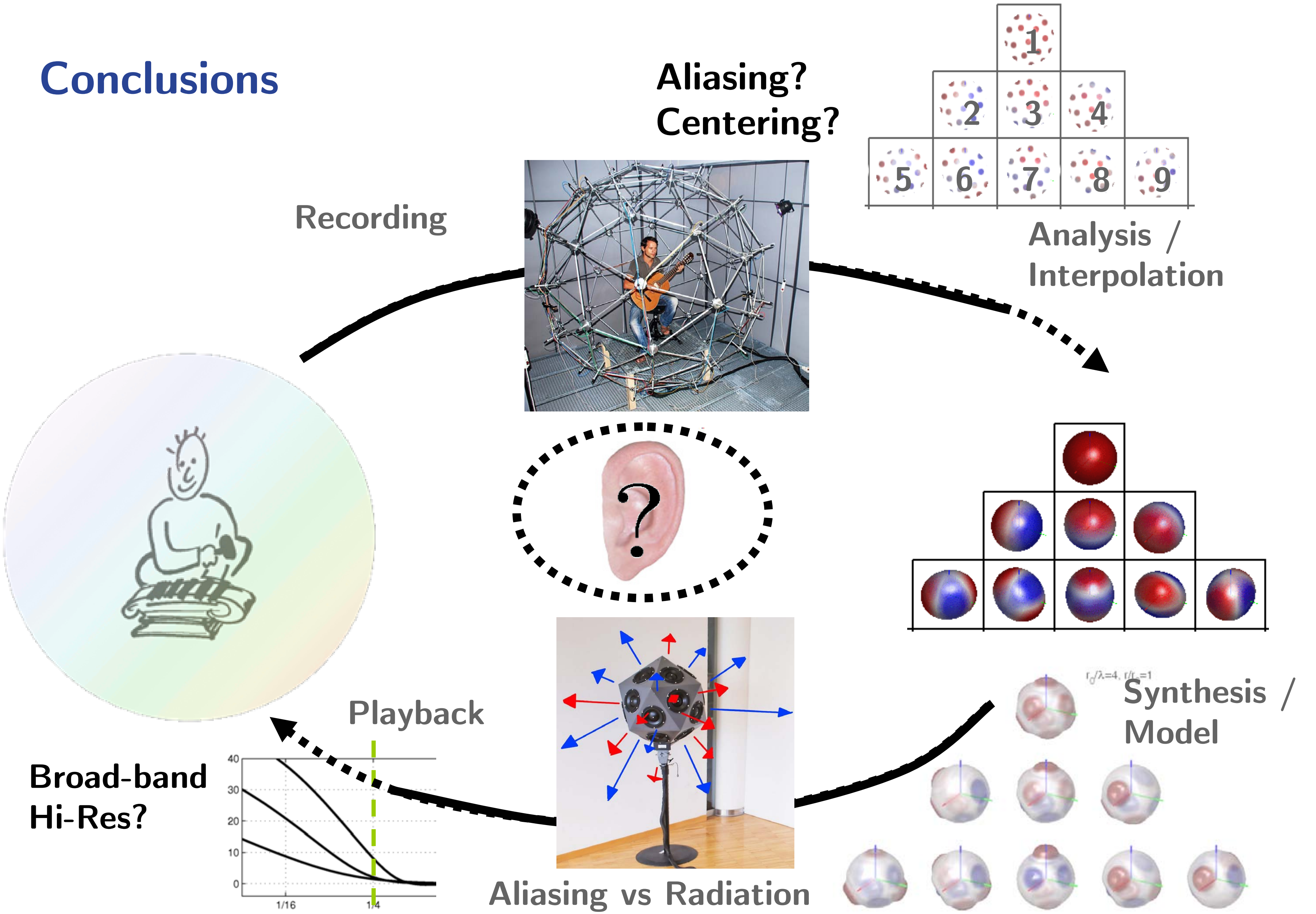


# Spherical Beam - DEMO

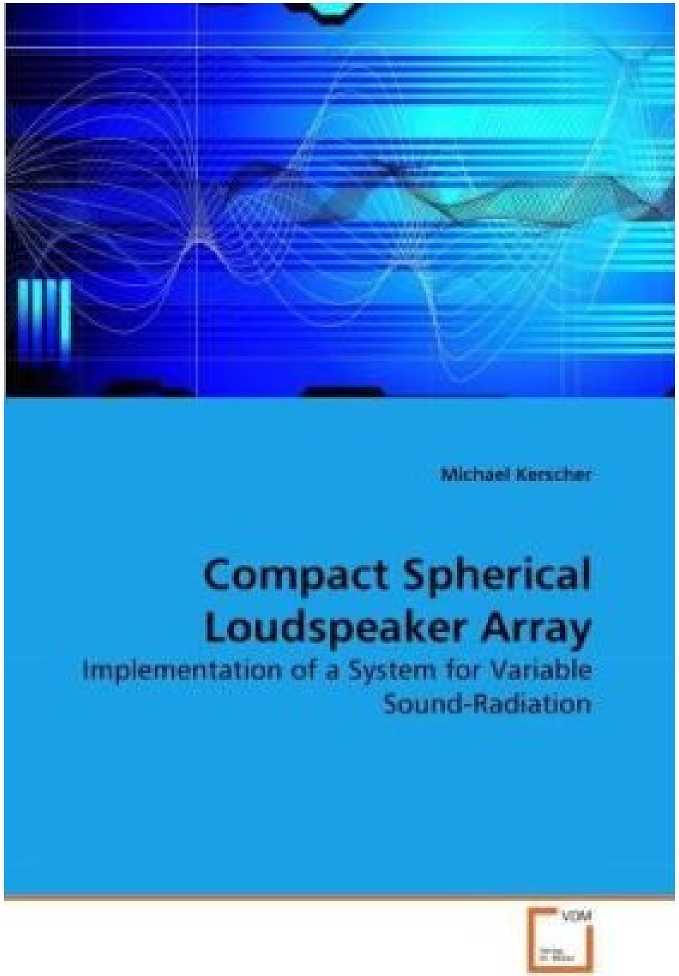




# Conclusions

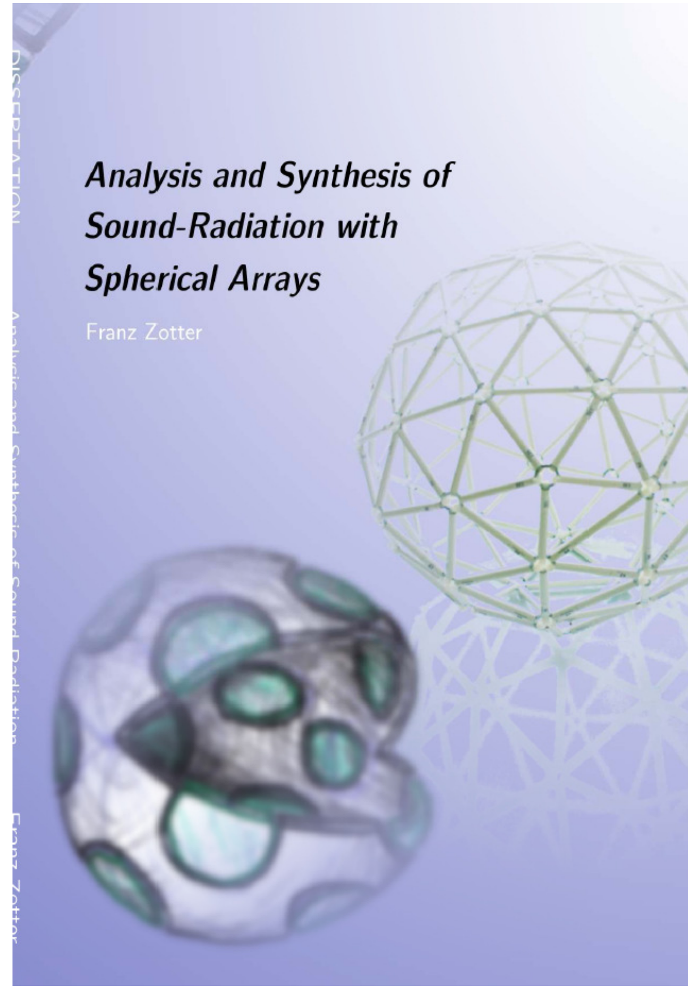


# Literature



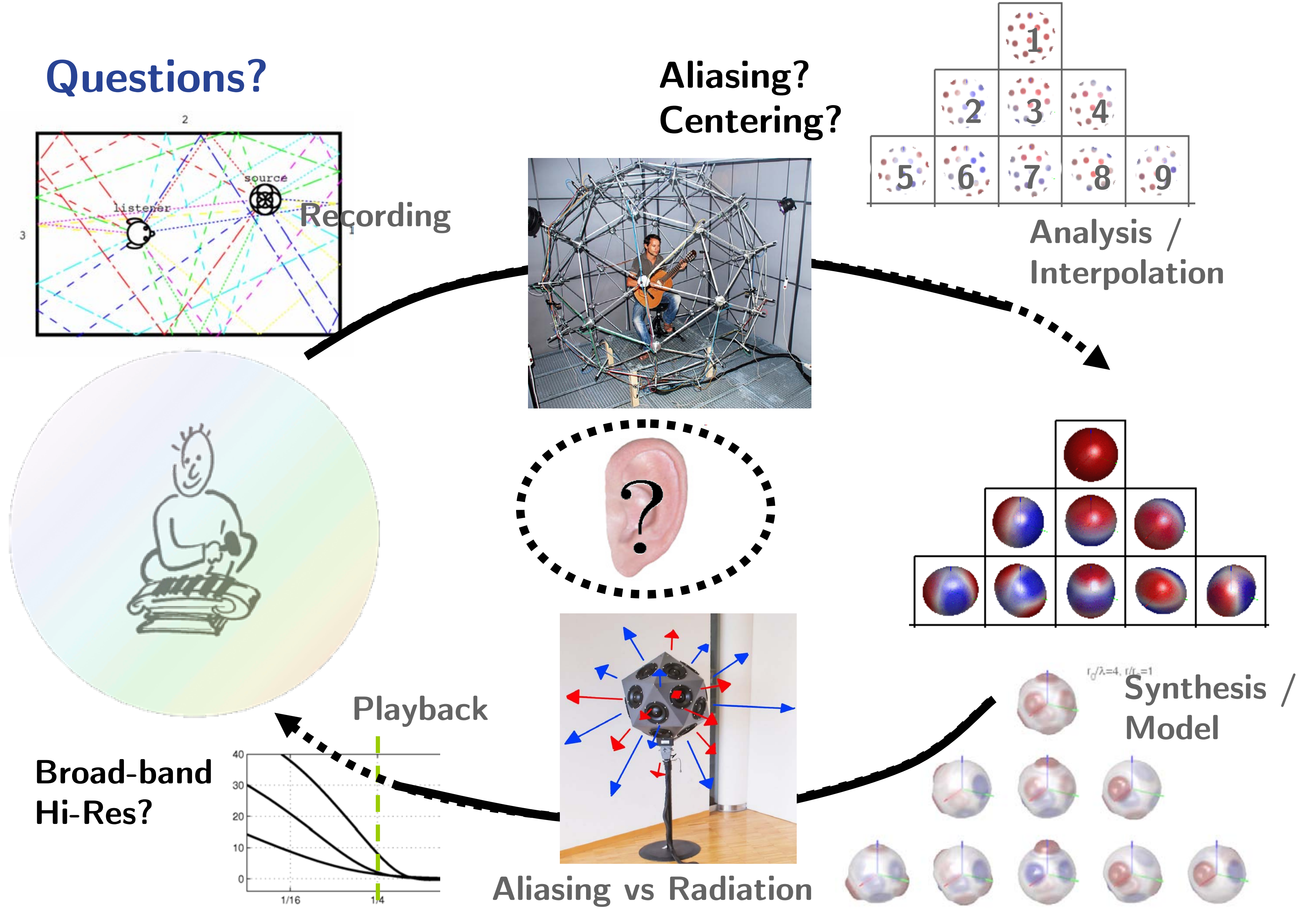

M. Thesis
Michael
Kerscher
(amazon)

PhD Thesis
Franz Zotter
(here)




And various things online under http://iem.at/Members/zotter

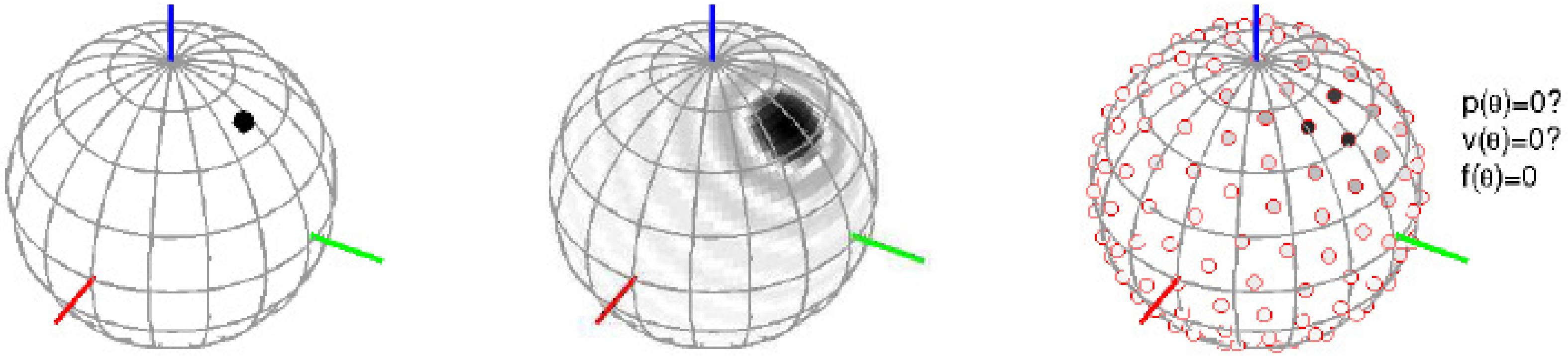
# Questions?


Aliasing?
Centering?
source
listener
Recording
1
2
3
4
5
6
7
8
9
Analysis / Interpolation
Synthesis / Model
Playback
Broad-band
Hi-Res?
40
30
20
10
0
1/16
1/4
Aliasing vs Radiation

# CUBE-DEMO

- You hear yourself

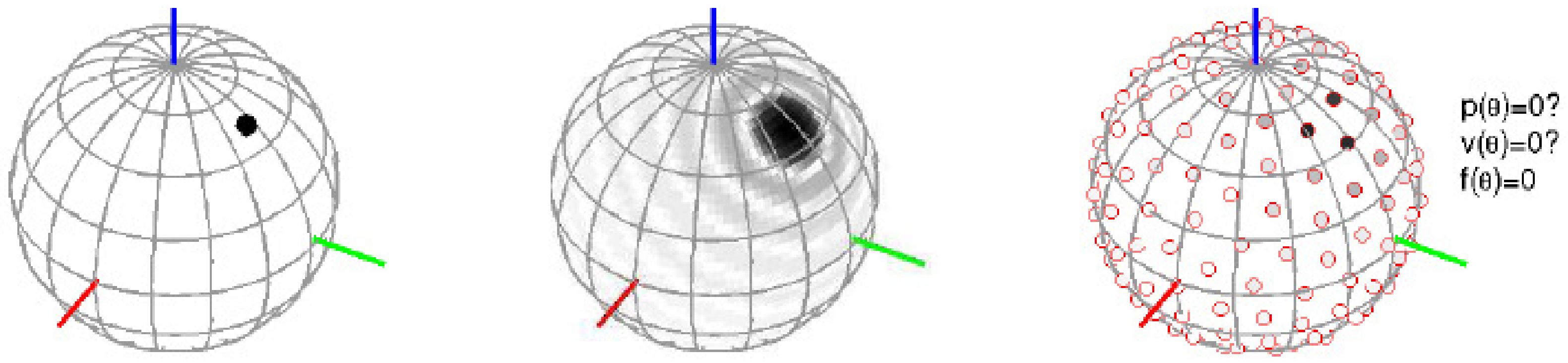
# Ambisonics Spatial Rendering



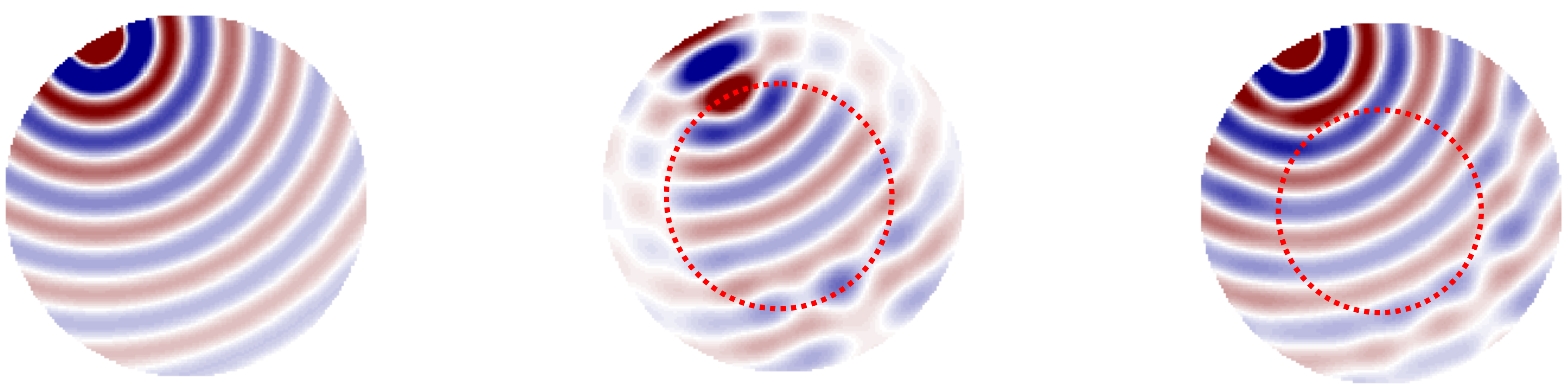

(a) Continuous distribution (b) Angularly band-limited distr. (c) Discretized distribution

„Order“ N:
relates to desired spatial smoothing and the number of available loudspeakers

# Ambisonics




(a) Continuous distribution (b) Angularly band-limited distr. (c) Discretized distribution



# *Verso la composizione eco-acustica*

Una veloce panoramica di tecniche e generi di produzione audio da suoni

ambientali...

**di David Monacchi**
www.davidmonacchi.it

From the environmental sound-art project: "Fragments of Extinction - Acoustic Biodiversity of the Primary Equatorial Rainforests" by David Monacchi

# resolution 1 (unsharp)

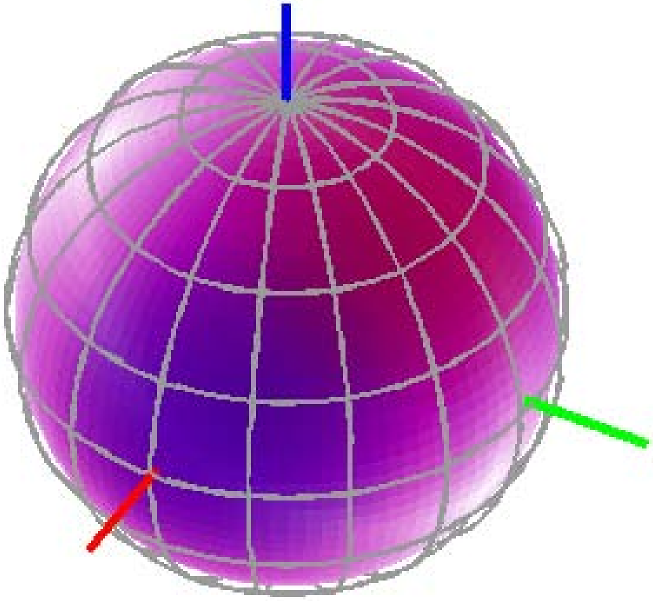


# resolution 2 (unsharp)



# resolution 3 (sharp)

## resolution on sphere



## increasing the resolution