DIGITAL EMULATION OF DISTORTION EFFECTS BY WAVE AND PHASE SHAPING METHODS

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ABSTRACT

This paper will consider wave (amplitude) and phase signal shaping techniques for the digital emulation of distortion effect processing. We examine how to determine the Wave- and Phase-shaping functions with harmonic amplitude and phase data. Three distortion effects units are used to provide test data. The action of the Wave- and Phase- shaping functions derived for these effects is demonstrated with the assistance of a super-resolution frequency-domain analysis technique.

1. INTRODUCTION

Distortion units essentially are a non-linear Waveshaping circuit that alters the amplitude of the input waveform thereby modifying its spectrum, typically using some form of diode-based clipping [1]. Digital emulation of these analogue processes has appeared in a variety of forms. In some, the aim is to directly model a specific analogue circuit [2], while in others the aim is to create algorithms that conceptually capture the analogue processing [3]. The advantages of the latter are flexibility and the ability to control the use of use of oversampling (a necessity for circuit modeling [1]). Emulation using the algorithmic approach can be divided into a nonlinear system model with memory or without memory [4]. However, incorporating memory for systems with strong nonlinearities is computationally expensive for real-time synthesis. Thus, it is more common to use a nonlinear system that is memoryless. While Waveshaping [5] is an amplitude distortion of the input signal, it has been shown more recently that distortion can also be applied to the signal phase to achieve a similar result [6], [7], [8]. In this work we establish a spectral connection between the Wave- and Phase- shaping methods of amplitude and phase distortion [9] respectively. To illustrate the analysis, examples of nonlinearly shaped sinusoids that have been processed using analogue distortion effects will be used.

The paper is organized as follows. We will first introduce and discuss the amplitude and phase signal shaping techniques. Then we will examine three distortion effects, followed by the application of the previously discussed algorithms to emulate these effects.

2. SIGNAL SHAPING METHODS

2.1. Non-linear Waveshaping

The basic idea is that given some sinewave at a frequency ω

$$x(t) = \cos(\omega t) \tag{1}$$

there is a nonlinear function *f*(.) that will alter the amplitude of *x*(*t*) to produce an output

$$y(t) = f\left(x(t)\right) = f\left(\cos(\omega t)\right) \tag{2}$$

In [10] Chebyshev polynomials were introduced as a useful description for such nonlinearities. The output of eq. 2 can be written as a power series

$$y(t) = d_0 + d_1 \cos(\omega t) + d_2 \cos^2(\omega t) + \ldots \tag{3}$$

or more compactly

$$y(t) = \sum_{p=0}^{\infty} d_p x^p(t) \tag{4}$$

Using a Fourier decomposition to determine the coefficients of eq.4 is difficult because of the expansion of the trigonometric product terms. The useful property of Chebyshev polynomials is that

$$T_k\left(\cos(\omega t)\right) = \cos(k\omega t) \tag{5}$$

where T_k denotes a Chebyshev polynomial of order *k*. Applying these polynomials to describe eq. 4 results in

$$y(t) = \frac{a_0}{2} T_0 + a_1 T_1 + a_2 T_2 + \ldots + a_N T_N \tag{6}$$

where a_0, a_1, a_2 ... are the Fourier series coefficient of *y(t)*.

The original work of [10] was extended by [11] to the synthesis of complex dynamic spectra. In particular, [11] provided a matrix based technique for computing the coefficients of the power series in eq.4 using the Fourier series coefficients of eq.6. This simplified the procedure for calculating the Waveshaping transfer function given a set of spectral harmonic magnitudes. Using an $(N+1)\times(N+1)$ generative matrix *P* the relationship is

$$\begin{bmatrix} d_0 \\ d_1 \\ \vdots \\ d_N \end{bmatrix} = 0.5 \begin{bmatrix} (2)^0 \\ (2)^1 \\ \vdots \\ (2)^N \end{bmatrix} P \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_N \end{bmatrix} \quad (7)$$

The first row of the generative matrix P is

$$p(1,j) = [1 \;\; 0 \;\; -2 \;\; 0\,2 \;\; 0 \;\; -2\ldots] \quad (8)$$

and subsequent rows can be computed using the recursion

$$p(i,j) = p(i-1, j-1) - p(i, j-2) \quad (9)$$

It can be noted from eq.7 that the harmonic phase is missing from the relationship. To include this means a phase quadrature form of Waveshaping [5]. With this, each harmonic magnitude, except the DC component, has an associated phase [5]. Defining these as

$$\boldsymbol{\varphi} = [\phi_0 \quad \phi_1 \; \phi_2 \ldots \phi_N] \quad (10)$$

To take account of the phase two Waveshaping polynomials now need to be generated. Furthermore, the second polynomial requires Chebyshev polynomials of the second kind. Again, following the example of [11] matrix relationships can be expressed for phase quadrature Waveshaping

$$\begin{bmatrix} dI_0 \\ dI_1 \\ \vdots \\ dI_N \end{bmatrix} = 0.5 \begin{bmatrix} (2)^0 \\ (2)^1 \\ \vdots \\ (2)^N \end{bmatrix} P \begin{bmatrix} a_0 \cos(-\phi_0) \\ a_1 \cos(-\phi_1) \\ \vdots \\ a_N \cos(-\phi_N) \end{bmatrix} \quad (11)$$

and

$$\begin{bmatrix} dQ_0 \\ \vdots \\ dQ_{N-1} \end{bmatrix} = 0.5 \begin{bmatrix} (2)^1 \\ \vdots \\ (2)^N \end{bmatrix} Q \begin{bmatrix} a_1 \sin(-\phi_1) \\ \vdots \\ a_N \sin(-\phi_N) \end{bmatrix} \quad (12)$$

where the first row of the generative matrix Q is

$$q(1,j) = [1 \;\; 0 \;\; -1 \;\; 0 1 \;\; 0 \;\; -1\ldots] \quad (13)$$

and subsequent rows can be computed using the recursion

$$q(i,j) = q(i-1, j-1) - q(i, j-2) \quad (14)$$

The quadrature waveshaper output is then given by

$$y(t) = \sum_{p=0}^{\infty} dI_p x^p(t) + \sin(\omega t) \sum_{p=0}^{\infty} dQ_p x^p(t) \quad (15)$$

2.2. Non-linear Phaseshaping

The technique of phase distortion, which is the core of our Phaseshaping approach, was originally proposed as a digital wave synthesis method [9]. More recently, in [7] and [8] it was explored as an efficient alternative for the design of virtual analogue oscillators, and in [6] it was proposed to implement it using time-varying allpass filters as a distortion effect on arbitrary inputs. To establish the connection between Wave- and Phaseshaping we start with the same sinusoidal signal of eq. 4, and propose that there is a non-linear function $g(.)$ such that

$$y(t) = \cos(g(\omega t)) \quad (16)$$

will produce a complex signal. While a detailed analysis of its spectrum is found in [8], it is also possible to establish a connection between the harmonic magnitudes of this signal and its instantaneous phase. First, defining a N harmonic analytic signal of fundamental frequency ω and amplitudes $a_1 \ldots a_N$ as

$$s(t) = a_1 e^{\omega t} + a_2 e^{2\omega t} + a_3 e^{3\omega t} + \ldots + a_N e^{N\omega t} \quad (17)$$

This can be written as into its real and imaginary components

$$s(t) = u(t) + jv(t) \quad (18)$$

where $u(t)$ and $v(t)$ denote the real and imaginary parts respectively. If the relationship between the harmonic magnitudes satisfies the conditions given in [12], then the instantaneous frequency of this signal can be written as [13]

$$\dot{\phi}_s(t) = \frac{u(t)\dot{v}(t) - \dot{u}(t)v(t)}{A^2} \quad (19)$$

where $\dot{u}(t)$ and $\dot{v}(t)$ are the first differences of the real and imaginary parts, respectively, and

$$A = \sqrt{u^2(t) + v^2(t)} \quad (20)$$

Substituting for $u(t)$, $v(t)$, $\dot{u}(t)$ and $\dot{v}(t)$ the above will lead to an expression from which the instantaneous frequency can be directly calculated.

Defining the combination

$$\mathbf{C} = \binom{N}{2} \quad (21)$$

This will have the set of combinadics denoted $\mathbf{M}_{(N,2)}$ that contains $L = N!/(2!(N-2)!)$ vectors each denoted as $M_{(N,2)}(.)$. Also, the vectors of magnitudes and frequencies are

$$\mathbf{a} = [a_1 a_2 \ldots a_N] \quad (22a)$$

and

$$\boldsymbol{\omega} = [\omega \; 2\omega \ldots N\omega] \quad (22b)$$

Then, defining two terms

$$\phi_{s\,num}(t) = \sum_{i=1}^{L} \mathbf{a}^2 . \boldsymbol{\omega} + \sum_{i=1}^{L} \boldsymbol{\omega}(M_{N,2}(i)) \mathbf{a}(M_{N,2}(i)) \cos(\boldsymbol{\omega}'(M_{N,2}(i))\, t) \quad (23a)$$

where $\boldsymbol{\omega}'$ is the difference between the pair of frequencies determined by $M_{(N,2)}(.)$ and

$$\phi_{s\,den}(t)=\sum_{i=1}^{L}\mathbf{a}^2+\sum_{i=1}^{L}2\,\mathbf{a}(M_{N,2}(i))\cos(\boldsymbol{\omega}'(M_{N,2}(i))\,t) \quad (23b)$$

The instantaneous frequency is given by

$$\dot{\phi}_s(t)=\frac{\dot{\phi}_{snum}(t)}{\dot{\phi}_{sden}(t)} \quad (24)$$

The instantaneous frequency can be written in terms of the fundamental frequency and frequency deviation

$$\dot{\phi}_s(t)=\omega+\dot{\phi}_{sdev}(t) \quad (25)$$

This deviation term can be integrated to convert it into an equivalent modulation or Phaseshaping

$$\phi_{s\,\mathrm{mod}}(t)=\sum_{n}\dot{\phi}_{sdev}(t) \quad (26)$$

Thus it is possible to represent a harmonic signal as the Phaseshaping of a cosine signal at the same fundamental frequency. No harmonic phases were taken into account so far and the equations need to be extended to achieve this. Using our phase vector defined in eq. 10, the terms in eq. 23a and 23b can be redefined

$$\phi_{s\,num}(t)=\sum_{i=1}^{L}\mathbf{a}^2.\boldsymbol{\omega}+ \sum_{i=1}^{L}\boldsymbol{\omega}(M_{N,2}(i))\mathbf{a}(M_{N,2}(i))\cos(\boldsymbol{\varphi}'(M_{N,2}(i))+\boldsymbol{\omega}'(M_{N,2}(i))\,t) \quad (27a)$$

where $\boldsymbol{\varphi}'$ is the difference between the pair of phases determined by $\boldsymbol{M}_{(N,2)}(.)$ and

$$\phi_{s\,den}(t)=\sum_{i=1}^{L}\mathbf{a}^2+ \sum_{i=1}^{L}2\,\mathbf{a}(M_{N,2}(i))\cos(\boldsymbol{\varphi}'(M_{N,2}(i))+\boldsymbol{\omega}'(M_{N,2}(i))\,t) \quad (27b)$$

These can be substituted into eq. 24, eq. 25 and eq. 26 to find the Phaseshaping that will produce the signal *s(t)*.

3. THE DISTORTION EFFECT

Two broad classes of distortion circuits exist: (a) Hard clipping and (b) Soft clipping [1]. For this work, three distortion circuits were used, built using PCBs purchased from [14]. The first was *El Griton*, a Tubescreamer-type circuit that gave an asymmetrical soft clipping. The second was *Disto-Uno*, a Boss DS-1 type circuit and the third was *MAS Distortion*, a MXR distortion-type circuit, both of which were hard clippers. These circuits will be termed as 'Overdrive', 'Clipper 1' and 'Clipper 2' in the following. The output waveform from all three circuits is shown in fig 1. The circuits were driven by a 2V peak-peak sinewave at frequency 146.8 Hz (note D) and the output was sampled at 44100Hz using an M-audio Audiophile soundcard. The three panels from top to bottom show the waveforms for the Overdrive, Clipper 1 and Clipper 2 respectively.

In fig. 1 the soft clipping action of the Overdrive contrasts to the hard clipping of the others as some of the roundness of the input sinewave is still visible in the output.

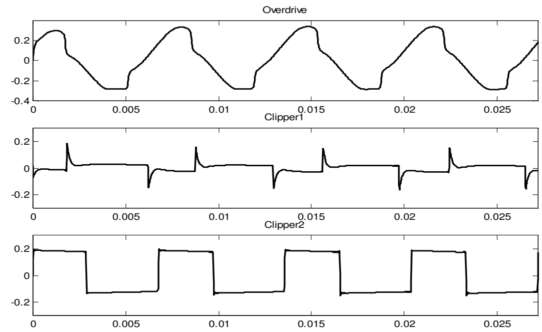

Figure 1: *Waveform outputs from the three distortion circuits fed by a sinewave signal.*

4. DIGITAL MODELLING BY MEANS OF WAVE- AND PHASE- SHAPING

Once the output waveforms were recorded the next task was to find the spectral peaks for input to the Wave- and Phase- shaping algorithms. A super-resolution frequency analysis technique was applied. Exact values for the harmonic magnitudes and phases of the measured waveforms were derived using the Complex Spectral Phase Evolution (CSPE) algorithm [15]. Empirically it was found that 40 harmonics only were required from the spectrum of the Overdrive for the analysis, otherwise the output of the waveshaper (eq. 4) was prone to instability. It was the weakness of the higher harmonics that gave rise to this. The magnitude of the 40th harmonic relative to the fundamental was below 60dB so the timbral reproduction should not be overly compromised.

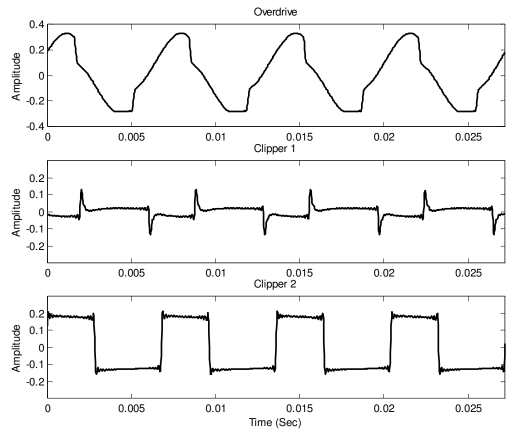

Figure 2: *Output of the phase quadrature waveshaper*

Using the phase quadrature Waveshaping with eq. 11, 12 and 15 produces the plot in fig. 2. The reproduction of the original waveshape can be achieved and compares well to the originals in fig. 1

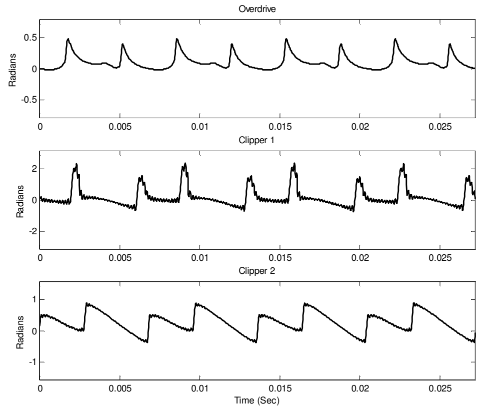

Figure 3: *Phaseshaping functions computing using harmonic magnitudes and phases*

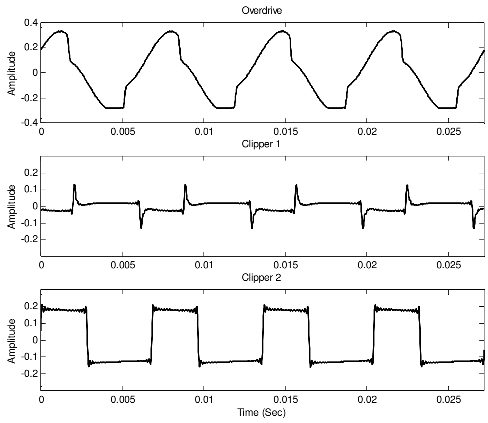

Figure 4 *Reconstructed waveforms using Phaseshaping functions derived from harmonic magnitudes and phases*

Fig. 3 plots the Phaseshaping functions when the phase information is included using eqs. 27a and 27b. Notably the phase excursion for the Clipper 1 is greatest whilst that for the Overdrive is smallest. Reconstructing the time waveforms from the Phaseshaping functions given in fig. 3 gives the result in fig. 4. Comparing the waveforms of fig. 4 with the originals in fig. 1 they have a good visual match. This demonstrates how amplitude and Phaseshaping can both produce equivalent results.

5. CONCLUSION

In this article, we have explored two techniques of signal shaping. Expressions were presented that extended the matrix approach of [11] to include quadrature Waveshaping, and also to compute a Phaseshaping function given a set of harmonic magnitudes and phases. These methods were applied to the emulation of distortion effects driven by a sinusoidal input. A good signal match was obtained utilizing with both the quadrature Waveshaping and Phaseshaping algorithms.

6. ACKNOWLEDGMENTS

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