STRING INSTRUMENT BODY MODELING USING FIR FILTER DESIGN AND AUTOREGRESSIVE PARAMETER ESTIMATION

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ABSTRACT

In this paper, a method is presented for modeling string instrument bodies. The crucial point of the method is a two-step filtering technique which combines advantages of both finite impulse response (FIR) filter design and autoregressive (AR) modeling. The frequency sampling method for FIR filter design enables modeling of specific resonance constellations within the magnitude spectrum, whereas a subsequent all-pole modeling step reduces the filter order. In addition, frequency warping is applied in order to further decrease the model complexity. The proposed method allows for highly-detailed modifications of individual resonances without affecting other resonances. An example for modifying a virtual violin body is presented as well as the implementation on a real-time platform which allows for experiments on perceived violin sound and musician-instrument interaction.

1. INTRODUCTION

The properties of a string instrument body are crucial for the quality of the instrument [1], [2]. The complex structure of resonances influences perceived timbre as well as specific directional radiation properties and reverberation characteristics [3], [4], [5], [6]. The body can approximately be considered as a linear time-invariant system and thus can be described by its impulse response. Usually, body impulse responses of string instruments are measured by exciting the instrument with dampened strings by an impulse or maximum length signal at the side of the bridge, e.g. in [3], [7], [8]. Fig. 1 shows a typical violin body impulse response. Fig. 2 shows the corresponding magnitude frequency response which, in this context, is referred to as the resonance profile.

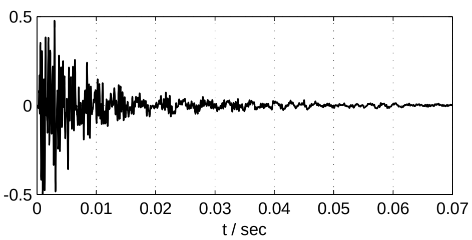

Figure 1: *Body impulse response of a violin.*

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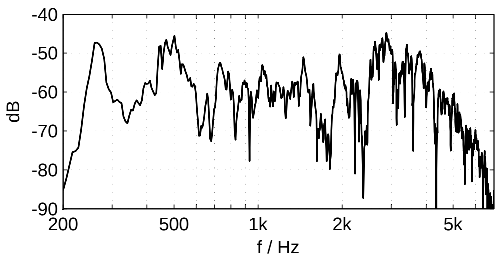

Figure 2: *Magnitude frequency response ('resonance profile') of a violin body.*

Modeling string instrument bodies is a frequently approached task. Previous studies have shown different methods to model resonance bodies, e.g. with electronic filter circuits [9] or digital filters [3], [10], [11], [12]. Most of the approaches concentrate either on computationally efficient modeling or on parameterizability, i.e. the ability to (virtually) modify instrument properties. Other works take into account the natural reverberation characteristics of a body [6], [13]. Also, warping techniques are used to match the frequency scale to the human auditory frequency resolution [6], [14]. Frequency warping reduces the high filter order which is required to accurately model the psychoacoustically important low-frequency resonances, e.g. the Helmholtz resonance or the main wood resonances.

This paper aims at extending state of the art body modeling approaches by a two-step filtering method which combines realistic sound properties with high parameterizability. The proposed method has been chosen with particular emphasis on the ability to intuitively modify body properties with high resolution on the basis of the magnitude frequency response. Using FIR filter design as well as autoregressive modeling and the above mentioned frequency warping technique, the method allows for simulating of specific resonance arrangements at lowest processing latency. In the context of this research project the authors aim to infer sound properties of violins from a real-time platform which consists of a silent violin and a virtual body synthesizer [15].

The paper is structured as follows: In Sec. 2, the filtering technique for modeling and modifying the body is described in detail. As an example, in Sec. 2.3, the technique is applied for modifying a violin resonance profile. Finally, in Sec. 3, a real-time implementation using MATLAB in combination with an external processor will be described briefly.

2. BODY IMPULSE RESPONSE MODELING USING FIR AND ALL-POLE FILTERS

It is evident that the most realistic sounding method to capture the complex resonance and reverberation characteristics of a string instrument is to make use of its original impulse response. The latter can serve as a starting point for further manipulations which, in turn, can be realized with digital filters. Due to the length of an impulse response (e.g. violin body: about 50 – 100 ms), this method causes a higher computational burden than synthesizing the whole body. However, in times of increased processor power this is well feasible (see also Sec. 3).

Concerning the specific application in this work, the filter technique has to fulfil two main requirements: (i) high resolution for modeling the low-frequency body modes and (ii) the possibility to define the desired filter response directly within the resonance profile. FIR filters, indeed, allow for direct frequency domain design but at the same time require a high filter order for a given accuracy. Due to the constant group delay of linear phase filters, this fact results in a perceptible latency which is non-acceptable in case of virtual musical instruments. Even an order of $N = 4096$ results in a group delay of 46 ms ($f_s = 44.1$ kHz) which is clearly noticeable by musicians. Using infinite impulse response (IIR) filters instead, leads to stability problems in case of direct frequency design and therefore is not practicable for high orders. Using stable parametric peak filters is problematic, too: Band interactions of the peak filters result in frequency response errors which have to be compensated for by computationally more expensive optimization algorithms [15].

The alternative solution which is presented here is based on the following two-step procedure (Sec. 2.1): First, the coefficients of a linear-phase FIR filter are computed by applying the frequency sampling method. Here, the filter order can be arbitrarily high in order to model the low-frequency modes with a desired resolution (e.g. $N = 8000$). In a second step, the coefficients of an AR model are estimated from the FIR filter coefficients using the *Yule-Walker* method. In doing so, the advantages of recursive and non-recursive filter design are combined: Due to the linear phase of the FIR filter, the frequency sampling method allows for modifications within the magnitude frequency response, whereas the AR modeling reduces the filter order and, at the same time, avoids a symmetric impulse response. Additionally applied frequency warping further reduces the filter order (Sec. 2.2).

2.1. Frequency Sampling and AR Modeling

In the present work, the resonance profile is modified by means of an arbitrary number of newly defined frequency samples within a magnitude spectrum plot (see also Sec. 3). The desired resonance profile $\left|H_d\left(e^{j\Omega}\right)\right|$ is computed afterwards using linear interpolation of the new frequency samples. Sampling the resonance profile at uniformly spaced frequency points leads to the complex frequency response

$$H_d\left(e^{j\Omega_k}\right) = \left|H_d\left(e^{j\Omega_k}\right)\right| \cdot e^{-j\frac{N_F-1}{2}\Omega_k}, \tag{1}$$

with the frequency points $\Omega_k = \frac{2\pi k}{N_F}$ and the frequency index $k = 0, 1, ..., N_F - 1$. N_F is the filter order. The linear phase is computed with [16]

$$e^{-j\frac{N_F-1}{2}\Omega_k} = e^{-j2\pi\frac{N_F-1}{2}\cdot\frac{k}{N_F}} \tag{2}$$

$$= \cos\left(2\pi\frac{N_F-1}{2}\cdot\frac{k}{N_F}\right) - j\cdot\sin\left(2\pi\frac{N_F-1}{2}\cdot\frac{k}{N_F}\right), \tag{3}$$

$$k = 0,\ 1,\ ...,\ \frac{N_F}{2} - 1. \tag{4}$$

The symmetric impulse response of the FIR filter is obtained after applying an N_F-point inverse DFT:

$$h_d(n) = \mathrm{IDFT}\left\{H_d\left(e^{j\Omega_k}\right)\right\} \tag{5}$$

$$= \frac{1}{N}\sum_{k=0}^{N_F-1} H_d\left(e^{j\Omega_k}\right)\cdot e^{j(2\pi n/N_F)\cdot k}. \tag{6}$$

In the second step, the parameters of an AR model are computed. In general, an AR model is an infinite impulse response filter which forms a white noise input signal to have similar spectral properties as a given signal [17]. In this case, the latter is the impulse response $h_d(n)$ of the FIR filter (Fig. 3).

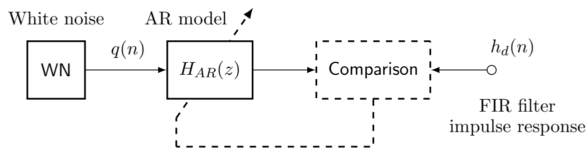

Figure 3: *Autoregressive model which forms a white noise input signal to have similar spectral properties as the FIR filter used for resonance modifications.*

The difference equation of the pth-order AR model can be written as

$$h_d(n) = q(n) - \sum_{i=1}^{p} a_i \cdot h_d(n-i), \tag{7}$$

where $h_d(n)$ is the output signal of the model, $q(n)$ is the white noise signal and a_i are the model parameters. The transfer function is given by

$$H_{AR}(z) = \frac{1}{1 + \sum_{i=1}^{p} a_i \cdot z^{-i}}. \tag{8}$$

Here, the *Yule-Walker* method (autocorrelation method) is used to determine the model parameters [17]. The autocorrelation sequence of the finite duration signal h_d (length N_F) is defined as

$$r_h(m) = \frac{1}{N_F}\cdot\sum_{n=0}^{N_F-1} h_d(n-m)\cdot h_d(n). \tag{9}$$

After setting (7) in (9) and considering the special spectral properties of the white noise source, the autocorrelation sequence can be written as

$$r_h(m) = \begin{cases} \sigma_Q^2 - \sum_{i=1}^{p} a_i \cdot r_h(i); & m = 0 \\ -\sum_{i=1}^{p} a_i \cdot r_h(m-i); & m > 0 \end{cases}, \quad (10)$$

where σ_Q^2 is the variance of the white noise. With the autocorrelation matrix

$$\mathbf{R}_h = \begin{bmatrix} r_h(0) & r_h(-1) & \cdots & r_h(-p) \\ r_h(1) & r_h(0) & \dots & r_h(-p+1) \\ \vdots & \vdots & \ddots & \vdots \\ r_h(p) & r_h(p-1) & \dots & r_h(0) \end{bmatrix}, \quad (11)$$

the autocorrelation vector

$$\mathbf{r}_h = [r_h(0), \ r_h(1), \ r_h(2), \ \cdots, \ r_h(p)]^T, \quad (12)$$

and the parameter vector

$$\mathbf{a} = [1, \ a_1, \ a_2, \ \cdots, \ a_p]^T, \quad (13)$$

(10) can be described as the linear matrix equation (*Yule-Walker* equation)

$$\mathbf{a} = -\mathbf{R}_h{}^{-1}\mathbf{r}_h. \quad (14)$$

Due to the Hermitian Toeplitz form of the autocorrelation matrix, the inversion in (14) can efficiently be done by means of the *Levinson-Durbin* recursion. Since the algorithm is described in many publications, e.g. in [17], details are omitted here. The solution of (14) leads to the set of model parameters, i.e. the coefficients of a stable all-pole system. The magnitude frequency response of the system approximates the desired resonance profile:

$$\left|H_{AR}\left(e^{j\Omega}\right)\right| \approx \left|H_d\left(e^{j\Omega}\right)\right|. \quad (15)$$

The precision of the approximation depends on the number of AR parameters p as well as on the FIR filter order N_F. It is obvious, that a small N_F results in a spectral smoothness which can not be compensated for by the AR model. In the present case, it turned out, that N_F and p have to be at least 8000 and 1500, respectively, for resulting in an adequate accuracy regarding the low-frequency body modes. Fig. 4 shows the resonance profile of an original violin body impulse response, the magnitude spectrum of the FIR filter ($N_F = 8000$), and the magnitude spectrum of the all-pole model ($p = 1500$). Since the order of the resulting all-pole model is still very high, frequency warping is applied. This is described in the following section.

2.2. Frequency Warping

As mentioned above, the order of the AR model can further be reduced by using frequency warping. Frequency warping allows a non-uniform frequency resolution which corresponds to the human auditory system [14], [18]. The warped frequency axis is computed with the all-pass phase function

$$\theta(\Omega) = \arctan \frac{\left(1-\lambda^2\right) \cdot \sin(\Omega)}{(1+\lambda^2)\cos(\Omega) - 2\lambda}, \quad (16)$$

where λ is the warping Parameter (*Laguerre* Parameter) and $\Omega = 2\pi f/f_S$. With $\lambda = 0.75$, the frequency axis is mapped to the auditory Bark scale (Fig. 5).

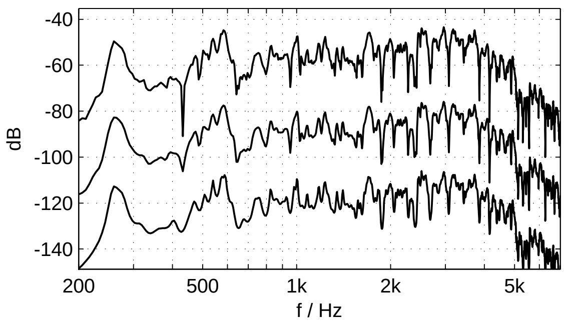

Figure 4: *From top down, separated with 30 dB offset: Original violin resonance profile, magnitude spectrum of the high-order FIR filter (order N_F = 8000), and magnitude spectrum of the corresponding high-order AR model (order p = 1500).*

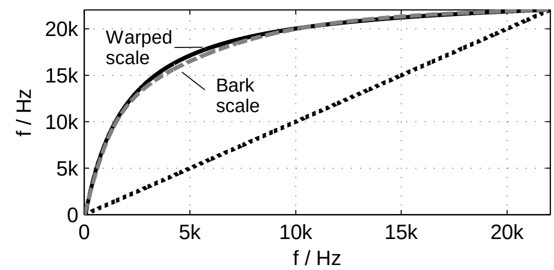

Figure 5: *Frequency scales: linear (dotted), Bark (Zwicker & Fastl, gray, dashed), and warped (λ = 0.75, black, solid).*

Using a warped frequency scale, the above described filter technique has to be slightly modified. Now, the desired magnitude frequency response $|H_d(e^{j\Omega})|$ is sampled on the warped scale instead of the equally spaced grid:

$$H_d\left(e^{j\Omega_k}\right) \rightarrow H'_d\left(e^{j\Omega'_k}\right), \quad (17)$$

with $\Omega'_k = \theta\left(\frac{2\pi k}{N_F T}\right) \cdot T$, $k = 0, 1, ..., N_F - 1$ and $T = 1/f_S$. The parameters of the AR model are estimated as described in Sec. 2.1, but now, the autocorrelation sequence of the 'warped' FIR filter impulse response is used. This leads to the transfer function $\bar{H}_{AR}(z)$. With the bilinear transformation [14]

$$\bar{z}^{-1} \rightarrow z^{-1} = \frac{z^{-1} + \lambda}{1 + \lambda z^{-1}} \quad (18)$$

applied to $\bar{H}_{AR}(z)$ with $\bar{h}_{AR}(n) = Z^{-1}\left\{\bar{H}_{AR}(z)\right\}$, the dewarped model transfer function is given by

$$H_{AR}(z) = \sum_{n=0}^{L-1} \bar{h}_{AR}(n) \cdot \bar{z}^{-n}. \quad (19)$$

L is the length of the impulse response $\bar{h}_{AR}(n)$. Directly implementing the dewarped all-pole filter yields to delay-free recursive loops. To avoid this, a modified filter structure is used which is described in [19]. Frequency warping reduces the filter order by a factor of about $6 - 10$. Fig. 6 shows the spectrum of the original impulse response in comparison with an AR model which is computed with frequency warping ($p = 250$).

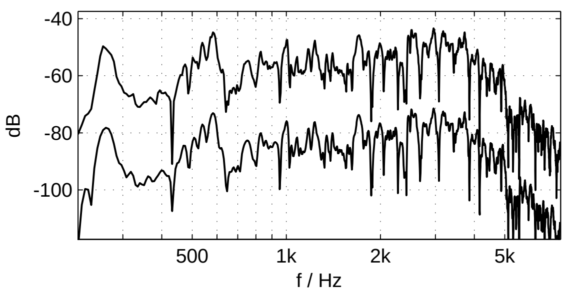

Figure 6: *Top: Spectrum of an AR model (order p = 1500) without frequency warping. Bottom (–30 dB): Spectrum of an AR model which is computed with frequency warping (order p = 250).*

2.3. Application Example

There are two options for applying the above described filter technique to an instrument body model (Fig. 7): a) The body is completely synthesized by an AR model. The desired resonance modifications are defined within the magnitude frequency response of the model. Frequency sampling leads to the FIR filter impulse response, afterwards, the new AR model is generated. b) The original body impulse response serves as starting point, changes are done in the resonance profile and afterwards, the difference of both, the original and the desired new spectrum (in dB) is computed. The resulting difference spectrum is the desired magnitude spectrum $\left|H_d\left(e^{j\Omega}\right)\right|$ of the modification filter. The filter is realized with the proposed two-step procedure and subsequently applied to the body impulse response. Method b) largely maintains the natural reverberant properties of a body apart from the frequency range which has been modified. The disadvantage of method b) has already been mentioned above: The realistic sound properties are obtained at the expense of computational cost which increases due to the length of the original body impulse response. Hence, in the present work, the filtering process is outsourced using an external signal processor (Sec. 3).

Both methods a) and b) allow for arbitrary modifications of individual resonances or complete resonance areas without affecting other resonances. Modifications can be boosting, cutting, shifting, broadening, generating of new resonances, etc. An example is shown in Fig. 8: modifications of a violin resonance profile in the range of the main corpus resonances (at about 450 - 550 Hz). This frequency range significantly affects the behaviour of the fundamental frequencies and therefore influences playability. The original resonance profile is shown as well as the desired new resonance profile which has been drawn within a graphical user interface (see Sec. 3). The AR model approximation (order $p = 250$) and the spectrum of the body impulse response modified with method b) are also shown. The modified impulse response in comparison to the original impulse response is shown in Fig. 9. Fig. 10 shows the difference spectrum, i.e. the desired frequency response of the modification filter, the magnitude frequency response of the corresponding all-pole filter, and its frequency-dependent error.

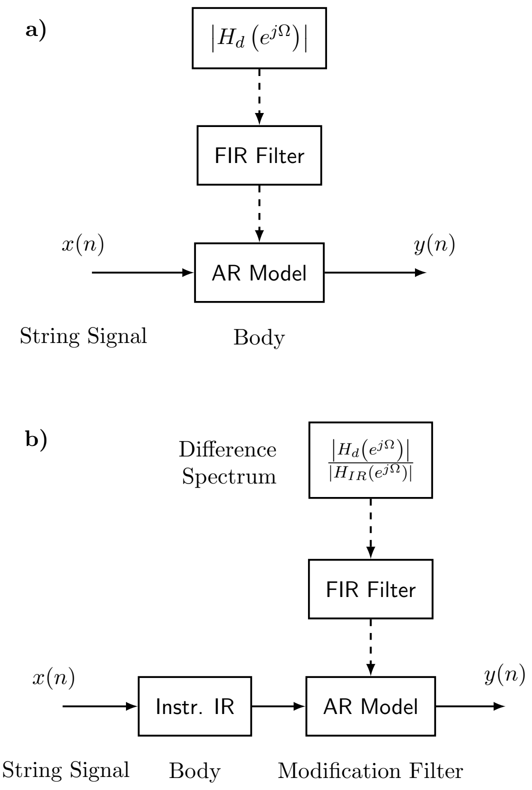

Figure 7: *The two body modeling options (see description in the text): a) The virtual instrument body is completely realized by an AR model which approximates the desired resonance profile $\left|H_d\left(e^{j\Omega}\right)\right|$. b) The difference spectrum between the desired resonance profile and the original resonance profile $\left|H_{IR}\left(e^{j\Omega}\right)\right|$ is the magnitude frequency response of a modification filter.*

3. IMPLEMENTATION

In the context of a research project on desirable violin sound properties, the virtual body filters the string signal of a silent generator violin. Fig. 11 shows the main components of the system. It is described in more detail in [15]. The filtering procedure is outsourced to a Texas Instruments TMS320C6416 DSP platform in order to achieve real-time sound processing. This results in an overall system latency of less than 5 ms which is sufficiently low for professional musicians. The resonance profile changes are defined directly within a magnitude frequency response plot in a MATLAB GUI (Fig. 12). The computations of the FIR filter coefficients and the AR model parameters are also done in MATLAB. The updated coefficients are sent to the external signal processor via real-time data exchange channels (RTDX). The degree of accuracy as well as the required computation load can be defined by adjusting the parameters N_F, p, and λ. Due to the efficient Levinson-Durbin algorithm, the estimation of the AR model parameters takes only little computation time which allows 'on-the-fly' modifications of the virtual body. This fact is particularly interesting for experiments with musicians where test subjects have to rate different body properties by direct comparison.

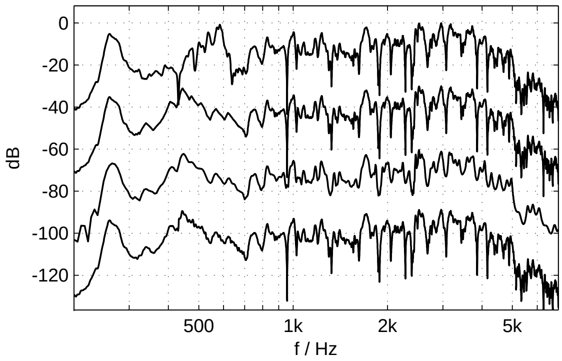

Figure 8: *Example of modification of the first violin corpus resonances. From top down, separated with 30 dB offset: Original violin resonance profile, desired new resonance profile, spectrum of the complete AR model, and spectrum of the modified impulse response (method b), see also description in the text and Fig. 7). In both cases, the AR order is p = 250.*

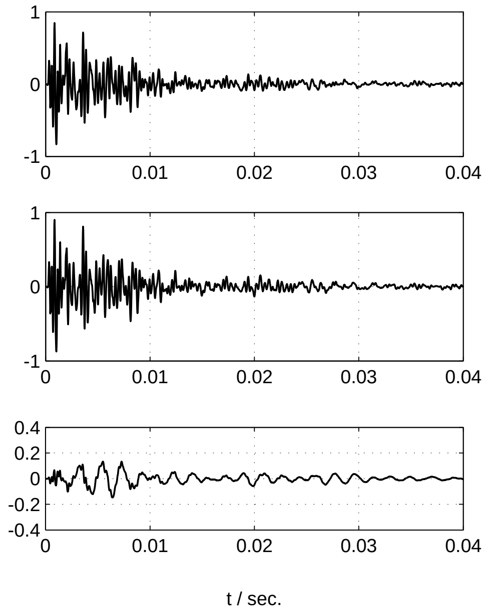

Figure 9: *Original violin impulse response (top) in comparison to the modified impulse response (middle, see example in the text). Bottom: difference plot.*

4. CONCLUSIONS

In this paper, a computational inexpensive filtering technique has been presented which, in the context of this research, is used to either model violin bodies or to modify given spectral properties of violin bodies. The method combines the advantages of FIR filter design and AR modeling. Thus, it allows for highly-detailed changes in the frequency domain while keeping the processing latency low. Individual resonances e.g. can be boosted or shifted without affecting other resonances. It has been shown that additionally applied frequency warping further reduces the filter order significantly. Also, an implementation within a real-time environment for testing sound properties of a violin has been described briefly.

Besides the application which is described here, the proposed filtering technique can also be used for other audio equalization tasks, e.g. in room acoustics.

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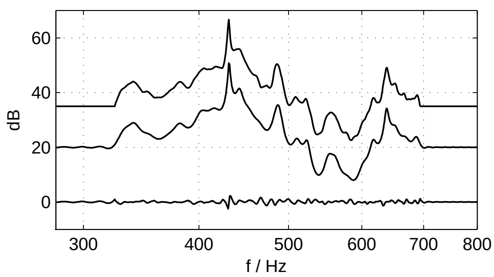

Figure 10: *From top down: Example difference spectrum (+35 dB offset, description in the text), spectrum of the modification filter (AR order p = 250, +20 dB offset), and the corresponding frequency-dependent error.*

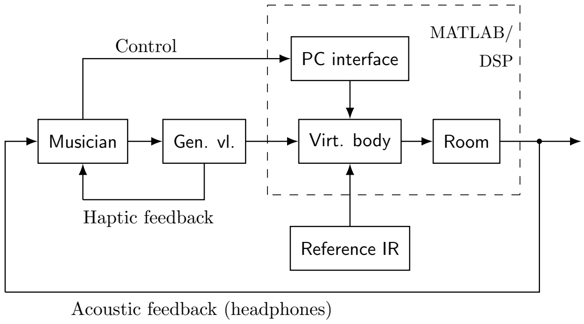

Figure 11: *Main components of the violin platform: the virtual body filters the string signals of a silent generator violin (Gen. vl.). Impulse responses of real violins (Reference IR) are used as starting point for further resonance modifications. The 'Room' block represents a standard reverberation effect.*

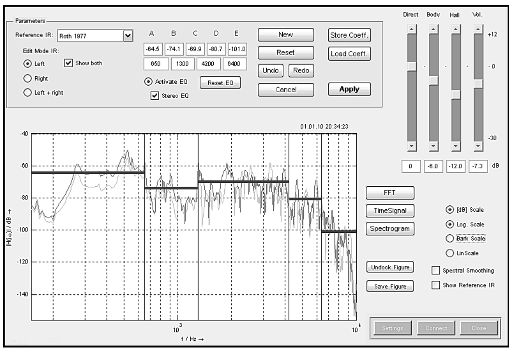

Figure 12: *MATLAB GUI with magnitude frequency response plot for defining modifications of resonance profiles. The horizontal lines in the spectrum plot represent an additional audio equalizer function [15].*

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