Spherical Arrays for Sound-Radiation Analysis and Synthesis

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Spherical Arrays for Analysis and Synthesis of Radiation

Hohl, Deboy, Zotter, 2009
Baumgartner, Messner, 2010

Zotter, Pomberger, 2007-2009
Zaar, Kößler 2009, Jochum, Reiner 2007
Pioneering Works

Domains that are interested in sound-radiation of Musical Instruments:

- Recording
- Room Acoustics
- Music
- Perception

artificially excited instrument
1966: wind instruments.

Ausg: 1972-2009
Pioneering Works

- Jürgen Meyer (1972)
- Weinreich and Arnold (1980)
- IRCAM (70s, Pierre Boulez)
- Franck Giron (1996)
- Cook and Trueman (1998)
- Olivier Warusfel et al. (1997-)
  La Timée
Remaining Issues

- Theoretical concept of sound-radiation recording and playback with spherical arrays

- Practical requirements, models, and processing ideas for sound-radiation

- Mathematical-physical basics for discrete recording, processing, and playback techniques

Note: Advantage of spherical microphone arrays direct capture instruments under musical excitation
Sound Radiation

- example:
  radiation „saron barung“
  (a Gamelan metallophone)

1st mic: frontal instrument height

2nd mic: frontal elevated
Sound Radiation

- Sound-source captured at 2 positions: bonang barung

→ two signals $x_1(t)$ and $x_2(t)$

$$x(\varphi, \vartheta, t) \ldots \text{sound-radiation signal}$$

$\varphi, \vartheta \ldots \text{angular coordinates}$
Theory of Sound-Radiation Analysis

Soap-Bubble Model

- Radial sound particle velocity

\[ x(\varphi, \vartheta, t) = v(\varphi, \vartheta, t) \]

Image: put musician and instrument into a big enough soap-bubble
Theory of Sound-Radiation Analysis
Soap-Bubble Model

- Radial sound particle velocity
  \[ x(\varphi, \vartheta, t) = v(\varphi, \vartheta, t) \]

Image: put musician and instrument into a big enough soap-bubble
Theory of Sound-Radiation Analysis
Surrounding Spherical Mic-Array

- Sound pressure

Image: put musician and instrument into a big enough mic-array

\[ x(\varphi, \vartheta, t) = p(\varphi, \vartheta, t) \]

\[ p(\varphi, \vartheta, t) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} Y_n^m(\varphi, \vartheta) \cdot \psi_{nm}(t) \]
Spherical Radiation Pattern

angular band-limit

\[ p_N(\varphi, \theta, \omega) = \sum_{n=0}^{N} \sum_{m=-n}^{n} Y_{nm}(\varphi, \theta) \cdot \psi_{nm}(\omega) \]
Spherical Harmonics

\[ Y_n^m(\varphi, \theta) = N_n^m P_n^m(\cos(\theta)) \begin{cases} \sin(m\varphi), & \text{for } m < 0 \\ \cos(m\varphi), & \text{for } m \geq 0 \end{cases} \]

\[ \begin{array}{cccccccc} m=-4 & m=-3 & m=-2 & m=-1 & m=0 & m=1 & m=2 & m=3 & m=4 \\ \hline n=0 & & & & \ast & & & & \\ n=1 & & & & & \ast & & & \\ n=2 & & & & & & \ast & & \\ n=3 & & & & & & & \ast & \\ n=4 & & & & & & & & \ast \end{array} \]
Theory of Sound-Radiation Analysis
Surrounding Spherical Mic-Array

- Sound pressure

Image: put musician and instrument into a big enough mic-array

How do we interpolate?

\[ p(t) = \begin{bmatrix} p(\varphi_1, \theta_1, t) \\ \vdots \\ p(\varphi_K, \theta_K, t) \end{bmatrix} \]
Decomposition of Sound-Radiation Signals

radiation pattern with finite angular bandwidth

\[
p_N(t) = \sum_{n=0}^{N} \sum_{m=-n}^{n} y^m_n \cdot \psi_{nm}(t)
\]

\[
y^m_n = \begin{bmatrix} Y^m_n(\varphi_1, \vartheta_1) \\ \vdots \\ Y^m_n(\varphi_K, \vartheta_K) \end{bmatrix}
\]

\begin{align*}
\text{m=-4} & & \text{m=-3} & & \text{m=-2} & & \text{m=-1} & & \text{m=0} & & \text{m=1} & & \text{m=2} & & \text{m=3} & & \text{m=4} \\
\end{align*}
Decomposition of Sound-Radiation Signals

radiation pattern with finite angular bandwidth

\[ p_N(t) = Y_N \cdot \psi_N(t) \]

\[ \psi^0 \cdot \psi^{-N} \cdot Y_N \cdot \psi_N(t) \]

\[ Y^m_n = \begin{bmatrix} Y^m_n(\varphi_1, \vartheta_1) \\ \vdots \\ Y^m_n(\varphi_K, \vartheta_K) \end{bmatrix} \]

\[ Y_N = \begin{bmatrix} y^0_0 & \cdots & y^N_{-N} & \cdots & y^0_N & \cdots & y^N_N \end{bmatrix} \]

\[ \psi^0_0(t) \]

\[ \vdots \]

\[ \psi^0_N(t) \]

\[ \psi^{-N}_N(t) \]

\[ \vdots \]

\[ \psi^N_N(t) \]

\[ = \begin{bmatrix} \psi^0_0(t) \\ \vdots \\ \psi^0_N(t) \\ \psi^{-N}_N(t) \\ \vdots \\ \psi^N_N(t) \end{bmatrix} \]
Decomposition of Sound-Radiation Signals

inversion yields coefficients

\[ p_N(t) = Y_N \cdot \psi_N(t) \]
\[ \psi_N(t) = Y_N^{-1} \cdot p(t) \]

This is how we interpolate (DEMO):

\[ p_N(\varphi, \vartheta, t) = \left[ Y_0^0(\varphi, \vartheta) \cdots Y_N^{-N}(\varphi, \vartheta) \cdots Y_N^0(\varphi, \vartheta) \cdots Y_N^N(\varphi, \vartheta) \right] \cdot \psi_N(t) \]

Nachbar, Nistelberger 2009.
Baumgartner, Messner 2010.
Compact Spherical Loudspeaker Arrays
Suitable for Holophony of Sound Radiation

Warusfel, Caussé, et al 1997 (FR)

Avizienis, Freed, Kassakian, Wessel, 2006
Schmeder 2009 (US)

Pasqual, Herzog, Arruda, 2008-2010 (BR/FR)

Behler, Witew, Pollow, 2006-2010 (DE)

Rafaely, Peleg, 2009-2010 (IL)

Zotter, Pomberger, Kerscher, 2007-2010 (AT)
Measuring the Directivity of a Compact Spherical Loudspeaker Array

Figure 1: Sampling Layout

Figure 2: Icosahedron and Semicircular Microphone Array
Directivity of a Compact Spherical Loudspeaker Array: An Acoustic MIMO-System

\[
\begin{bmatrix}
p_1(\omega) \\
p_2(\omega) \\
\vdots \\
p_K(\omega)
\end{bmatrix} =
\begin{bmatrix}
G_{11}(\omega) & \cdots & G_{1L}(\omega) \\
\vdots & \ddots & \vdots \\
G_{K1}(\omega) & \cdots & G_{KL}(\omega)
\end{bmatrix}
\cdot
\begin{bmatrix}
u_1(\omega) \\
v_2(\omega) \\
\vdots \\
v_L(\omega)
\end{bmatrix}
\]

\( K > L \ldots \) microphones
\( L \ldots \) loudspeakers
Directivity of One Array-Speaker

\[ p_1(\omega) = G(\omega) \cdot \begin{bmatrix} 1 \\ 0 \\ \vdots \end{bmatrix} \]
Directivity of a Compact Spherical Loudspeaker Array: An Acoustic MIMO-System

\[ p(\omega) \approx \hat{p}(\omega) \]

\[ p(\omega) = G(\omega) \cdot u(\omega) \]

\[ u(\omega) = G^{-1}(\omega) \cdot \hat{p}(\omega) \]
Different Angular Array-Layouts?

Lokki, 2008

Pollow, Behler, Reuter, 2008
Directivity of a Compact Spherical Loudspeaker Array: An Acoustic MIMO-System

\[ \psi_N(\omega) = Y_N^{-1} G(\omega) \cdot u(\omega) \]
Directivity of a Compact Spherical Loudspeaker Array: An Acoustic MIMO-System

\[ \psi_N(\omega) = Y_N^{-1} G(\omega) \cdot u(\omega) \]

\[ u(\omega) = G^{-1}(\omega) Y_N \cdot \gamma_N(\omega) \]

\[ \psi_N(\omega) \approx \gamma_N(\omega) \]
Directivity of a Compact Spherical Loudspeaker Array: An Acoustic MIMO-System

- Array Design
- Error range / control
- Dependency on Frequency
- (Acoustics)

\[
\psi_N(\omega) = Y_N^{-1} G(\omega) \cdot u(\omega)
\]

\[
u(\omega) = G^{-1}(\omega) Y_N \cdot \gamma_N(\omega)
\]
Compact Spherical Loudspeaker Array Surface $r_0$

$$
\nu_{nm} = \int_{S^2} v(\varphi, \vartheta) Y_n^m(\varphi, \vartheta) \, dS
$$

$$
v(\varphi, \vartheta) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} Y_n^m(\varphi, \vartheta) \cdot \nu_{nm}
$$
Compact Spherical Loudspeaker Array Surface $r_0$

$$\eta(\varphi, \theta) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} Y_n^m(\varphi, \theta) \cdot \nu_{nm}$$

$N=25$
Compact Spherical Loudspeaker Array Surface $r_0$

$$v(\varphi, \theta) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} y_n^m(\varphi, \theta) \cdot \nu_{nm}$$

$N=25$ physical shape / ALIASING

$$N_{\text{ctl}} = 2$$

$$L = (N_{\text{ctl}} + 1)^2 = 9$$
Compact Spherical Loudspeaker Array Surface $r_0$

$$v_5(\varphi, \psi) = \sum_{n=0}^{5} \sum_{m=-n}^{n} Y_n^m(\varphi, \psi) \cdot \nu_{nm}$$

$N=5$

unphysical attempt: ALIASING SUPPRESSION

$$N_{ctl} = 2$$
$$L = (N_{ctl} + 1)^2 = 9$$
Compact Spherical Loudspeaker Array Surface $r_0$

$v_2(\varphi, \theta) = \sum_{n=0}^{2} \sum_{m=-n}^{n} Y_n^m(\varphi, \theta) \cdot \nu_{nm}$

$N=2$

But: How?

General Rule:

$N_{\text{ctl}} \leq \sqrt{L} - 1$

$N_{\text{ctl}} \ldots$ cutoff-order

$L \ldots$ loudspeakers
Compact Spherical Loudspeaker Array Radiation

- Spherical Boundary Value Problem (Neumann)

Helmholtz-equation (ac.wv.)

\[
(k^2 + \Delta) p = 0
\]

Laplace-operator (spherical coordinates, chain rule)

\[
\Delta_{r, \varphi, \theta} p = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial p}{\partial r} \right) + \frac{1}{r^2 \sin(\vartheta)} \frac{\partial}{\partial \vartheta} \left( \sin(\vartheta) \frac{\partial p}{\partial \vartheta} \right) + \frac{1}{r^2 \sin(\vartheta)} \frac{\partial^2 p}{\partial \varphi^2}
\]
\[ v(\varphi, \vartheta)|_{r_0} = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} Y_n^m(\varphi, \vartheta) \cdot \nu_{nm}|_{r_0} \]

\[ p(kr, \varphi, \vartheta) = -i \rho_0 c \sum_{n=0}^{\infty} \sum_{m=-n}^{n} Y_n^m(\varphi, \vartheta) \left( \frac{h_n(kr)}{h'_n(kr_0)} \right) \nu_{nm}|_{r_0} \]

Small array / long wave (lo f)
\[ \frac{r_0}{\lambda} = 1 \]

Large array / short wave (hi f)
\[ \frac{r_0}{\lambda} = 4 \]

On surface
\[ \frac{r}{r_0} = 1 \]

In air
\[ \frac{r}{r_0} = 4 \]

\[ kr_0 = 2\pi \frac{r_0}{\lambda} \]
\[ v_N(\varphi, \theta) = \sum_{n=0}^{N} \sum_{m=-n}^{n} Y_n^m(\varphi, \theta) \cdot \nu_{nm} \]

\[ p(kr, \varphi, \theta) = -i \rho_0 c \sum_{n=0}^{\infty} \sum_{m=-n}^{n} Y_n^m(\varphi, \theta) \left( \frac{h_n(kr)}{h_n'(kr_0)} \right) \nu_{nm} \bigg|_{r_0} \]

\[ r_0/\lambda=0.25, \ r/r_0=1 \]

long wave (lo f)

on surface
\[ v_N(\varphi, \theta) = \sum_{n=0}^{N} \sum_{m=-n}^{n} Y_n^m(\varphi, \theta) \cdot \nu_{nm} \]

\[ p(kr, \varphi, \theta) = -i \rho_0 c \sum_{n=0}^{\infty} \sum_{m=-n}^{n} Y_n^m(\varphi, \theta) \frac{h_n(kr)}{h'_n(kr_0)} \nu_{nm} \bigg|_{r_0} \]

\[ r_0/\lambda = 0.25, \quad r/r_0 = 4 \]

The diagram illustrates long waves in air.
\[ v_N(\varphi, \vartheta) = \sum_{n=0}^{N} \sum_{m=-n}^{n} Y_n^m(\varphi, \vartheta) \cdot \nu_{nm} \]

\[ p(kr, \varphi, \vartheta) = -i \rho_0 c \sum_{n=0}^{\infty} \sum_{m=-n}^{n} Y_n^m(\varphi, \vartheta) \left( \frac{h_n(kr)}{h_n'(kr_0)} \right) \nu_{nm} \Bigg|_{r_0} \]

\[ r_0/\lambda = 4, \ r/r_0 = 1 \]

short wave (hi f)

on surface
\[ v_N(\varphi, \vartheta) = \sum_{n=0}^{N} \sum_{m=-n}^{n} Y_n^m(\varphi, \vartheta) \cdot \nu_{nm} \]

\[ p(kr, \varphi, \vartheta) = -i \rho_0 c \sum_{n=0}^{\infty} \sum_{m=-n}^{n} Y_n^m(\varphi, \vartheta) \frac{h_n(kr)}{h'_n(kr_0)} \nu_{nm} \bigg|_{r_0} \]

\( r_0/\lambda = 4, \ t/r_0 = 4 \)

short wave (hi f)

in air
Radial Propagation Term / Controllability

\[
\frac{h_n(8\pi r_0/\lambda)}{h'_n(2\pi r_0/\lambda)}
\]

in air

Controllability diagram showing dB vs. \( r_0/\lambda \) for different values of \( n \). The controlled region is indicated for \( n \leq \sqrt{L} - 1 \).
Radial Propagation Term / Aliasing

\[
\frac{h_n(8\pi r_0/\lambda)}{h'_n(2\pi r_0/\lambda)}
\]

in air

controlled
\[n \leq \sqrt{L} - 1\]

unsuppressed aliasing

\[\text{dB}\]

\[n = 0\]
\[n = 1\]
\[n = 2\]

controlled

uncontrolled

\[r_0/\lambda \propto f\]

lo f

hi f
Radial Propagation Term / Frequency Ranges for $N = 2$

\[
\left| \frac{h_n(8\pi r_0 / \lambda)}{h'_n(2\pi r_0 / \lambda)} \right|^{-1}
\]

in air

![Graph showing dB vs. $r_0 / \lambda$ for different $n$ values](image)
Different Array-Radii 😊
Spherical Beam - DEMO
Conclusions

Recording

Aliasing? Centering?

Analysis / Interpolation

Aliasing vs Radiation

Broad-band Hi-Res?

Playback
Literature

M. Thesis
Michael Kerscher
(amazon)

PhD Thesis
Franz Zotter
(here)

And various things online under http://iem.at/Members/zotter
CUBE-DEMO

- You hear yourself
Ambisonics Spatial Rendering

“Order“ N:
relates to desired spatial smoothing and the number of available loudspeakers
Ambisonics

(a) Continuous distribution  (b) Angularly band-limited distr.  (c) Discretized distribution
Verso la composizione eco-acustica

Una veloce panoramica di tecniche e generi di produzione audio da suoni ambientali...

di David Monacchi
www.davidmonacchi.it

From the environmental sound-art project: "Fragments of Extinction - Acoustic Biodiversity of the Primary Equatorial Rainforests" by David Monacchi
resolution 1 (unsharp)
resolution 2 (unsharp)
resolution 3 (sharp)
resolution on sphere
increasing the resolution